

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. [6 points] Please answer **two** of the following three questions.
 - (a) The Solow (1956) model is usually classified as an exogenous growth model. Present its main characteristics and justify that classification. Which features of modern growth are captured and not captured by that model ?
 - (b) In simple expansion of varieties growth models the aggregate production function takes the form of $Y(t) = AN(t)$, where N is the number of goods (or industries). Provide a description of the underlying assumptions leading to that production function. Explain how economic growth can occur according to those models ?
 - (c) The directed technical change model intends to explain the puzzling behavior of the relative wage for skilled relative to non-skilled labor in modern economies. Which puzzle is that ? By presenting the main assumptions of the model, describe how it provides an explanation for the puzzle.
2. [7 points] Let the dynamics of population be described by the differential equation $\dot{L} = (b - m)L$, where L is total population. Assume that both the birth and the mortality rates are functions of per capita GDP, y , where $b = \beta y$ and $m = \mu/y$, for $\beta > 0$ and $\mu > 0$. The technology for this economy is represented by the production function $Y = X^\alpha L^{1-\alpha}$ where $0 < \alpha < 1$, and X denoted the land endowment. The initial population is given, $L(0) = L_0$.
 - (a) Write an equation for the population dynamics.
 - (b) Determine the steady state level of population and study their dynamic behaviour.
 - (c) What are the effects of a reduction in the mortality rate parameter μ , on population and the per capita GDP ?
 - (d) Does long run growth exists for this economy ? Provide an economic intuition.
3. [7 points] Consider a centralized economy in which the representative consumer has the intertemporal utility function

$$\max_{\{C\}_{t \geq 0}} \int_0^\infty \frac{1}{1-\theta} C(t)^{1-\theta} e^{-\rho t} dt$$

where $\theta > 0$ and $\rho > 0$, and the aggregate economy constraint is

$$\dot{K} = AK(t)^\alpha - C(t)$$

where $0 < \alpha < 1$:

- (a) Do the necessary conditions for the existence of a balanced growth path, with a positive growth rate, hold ? Justify.
- (b) Consider a decentralized economy in which the problem of the representative agent is as before, but the productivity verifies the equation $A = A_0(K^a)^\beta$, where K^a is the aggregate level of capital for the economy, which is taken as exogenous by the consumer. Obtain a representation of the general equilibrium as a dynamical system in (K, C) . (Consider this economy from now on.)

- (c) Under which conditions a balanced growth path exists ? Assume, from now on, the conditions you arrived at. In this case will we have transitional dynamics ? Why ?
- (d) Is that general equilibrium Pareto efficient ? Why ?
- (e) In the previous decentralized economy, introduce the government fiscal policy, such that the budget is balanced at all times. Under which conditions can the fiscal policy make the general equilibrium of the economy efficient ?