Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Growth Economics
2019-2020

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Exam. **Época Normal** (First exam)
4.6.2020
First part: 18.00h-19.00h

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
  1. In your answer to questions (a) and (b), please start with a **very short explanation** of your reasoning.
  2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Points: 1(a) - 1.5, 1(b) - 1.5 , 1(c) - 2, 2(a) - 1.5, 2(b) -1.5, and 2(c) - 2.
- Your exam will only be considered if it is uploaded in Aquila between 19:00 and 19:05.

Part 1 Consider an economy in which the aggregate production function is Cobb-Douglas $Y(t) = AX^{\beta}L(t)^{1-\beta}$, with $0 < \beta < 1$, where $A > 0$ is the total factor productivity, X is the stock of land and $L(t)$ and $Y(t)$ are the level of population and the aggregate output at time $t \geq 0$. There is population growth according to the equation

$$\dot{L} = \big(b - m\big)\, L,$$

where the mortality rate, $m > 0$, is constant and exogenous and the fertility rate, b, is endogenous. The fertility rate is determined from the solution of the representative farmer problem: $\max_{c,b}\{u(c,b) : \ c + \mu b \leq y\}$ where c and y denote per-capita consumption and income, and μ is the unit cost of raising children. Assume that the farmer's utility function is

$$u(c,b) = \frac{\big(c\, b^{\theta}\big)^{1-\sigma} - 1}{1-\sigma},\ \theta > 0,\ \sigma > 0.$$

(a) Obtain the equation representing population dynamics.

(b) Solve (explicitly) the previous equation, assuming the initial population level $L(0) = L_0$ is given.

(c) Charaterize the behavior of per capita income in this economy.

Part 2 In the previous economy, consider that there is a benevolent monarch which wants to find an optimal path for population by using aggregate consumption, C, as a control variable. The utility function of the monarch is

$$\int_0^\infty \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to the same law of motion for L, as in Part 1, such that

$$b\, L(t) = \frac{Y(t) - C(t)}{\mu}.$$

Assume again that the initial population level $L(0) = L_0$ is given and that the population is asymptotically bounded by $\lim_{t\to\infty} L(t)e^{-\rho t} \geq 0$.

(a) Obtain the MHDS (maximized Hamiltonian dynamic system) for this problem in the (L, C) space, together with the initial and transversality conditions.

(b) Draw the phase diagram and discuss its properties.

(c) Charaterize the long-run behavior of per capita income in this economy. Discuss your result by comparing it with your answer to Part 1 (c).