Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Growth Economics
2019-2020

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Exam. **Época Normal** (First exam)
4.6.2020
Second part: 19.10h-20.10h

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
  1. In your answer to questions (a) and (b), please start with a **very short explanation** of your reasoning.
  2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Points: 1(a) - 2, 1(b) - 2, 1(c) - 1, 2(a) - 1, 2(b) - 3, and 2(c) - 1.
- Your exam will only be considered if it is uploaded in Aquila between 20:10 and 20:15.

Part 1 Consider a Solow economy in which the production technology uses machines, labor and robots. While machines are imperfectly substitutable with labor and robots, robots and labor are perfectly substitutable. This basically means that a machine (ex a truck) can be driven by a human or a robot. In particular let aggregate output, Y, be produced using the production function

$$Y(t) = K(t)^{\alpha}\Big(L(t) + M(t)\Big)^{1-\alpha}, \text{ with } 0 < \alpha < 1,$$

where K, L and M denote machines, labor and robots, respectively. There is full employment and population grows at a rate $n > 0$. The initial levels of machines, population and robots are all given and positive, v.g. $K(0) = k_0 > 0$, $L(0) = l_0 > 0$ and $M(0) = m_0 > 0$. Furthermore, assume: (H1) savings finance gross investment in machines, i.e.$S(t) = \dot{K} + \delta K(t)$, where $\delta > 0$ is the depreciation rate, and $S(t) = s\,Y(t)$, with $0 < s < 1$; and (H2) robots grow exogenously at the growth rate $n > 0$.

(a) Denoting the per-capita output by $y = Y/L$, obtain a differential equation displaying the dynamics of y.

(b) Solve the linearized equation for y in the neighborhood of a (positivelly valued) steady state.

(c) Is there long-run growth in this economy ? Charaterize the behavior of per capita income in this economy, and in particular the consequences from increasing m_0.

Part 2 In the previous economy, consider all the previous assumptions, except H1 and H2. Assume instead that savings finance not only gross investments in machines but also investments in robots. If the proportion of savings financing machines is denoted by ρ, such that $0 < \rho < 1$, we have $\rho\, S(t) = \dot{K} \; + \delta K(t)$ and $(1-\rho)\, S(t) = \dot{M} \; + \delta M(t)$.

(a) Let $k = K/L$ and $m = M/L$ be the intensities of machines and robots. Obtain a differential equation system for the joint dynamics of k and m.

(b) Assume that $(1-\rho)\, k_0 = \rho\, m_0$. Prove that linearizing the solution in the neighborhood of the steady state we obtain

$$k(t) = \frac{\tilde{k}}{1-\tilde{m}} + \left(k_0 - \frac{\tilde{k}}{1-\tilde{m}}\right) e^{\gamma t}, \; t \in [0, \infty)$$
$$m(t) = \left(\frac{1-\rho}{\rho}\right) k(t), \; t \in [0, \infty)$$

where $\tilde{k} \equiv \left(\frac{s\rho}{n+\delta}\right)^{\frac{1}{1-\alpha}}$, $\tilde{m} \equiv \left(\frac{1-\rho}{\rho}\right) \tilde{k}$ and $\gamma \equiv (1-\alpha)(n+\delta)(\tilde{m}-1)$.

(c) Assume $\tilde{m} > 1$. Charaterize the long-run behavior of per capita income in this economy. Discuss your result by comparing it with what you have obtained in Part 1 (c).