Growth, taxes and public debt

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The state and growth

- We can see the **state** as a sector which has the "technology" of changing aggregate allocations.
- This covers a big number of issues: setting up an institutional framework allocating asset and income rights, meaning that it can also change the allocation of assets and income, producing or supplying public goods, pooling information, changing adjustments cost of several type, etc;
- However, when its intervention generates pecuniary costs it should finance those costs, which means again that, given its size, it will imply changes in incentives.

Brief historical evidence

- The construction of the fiscal state has been historically related to war financing (an episode for Portugal 1640)
- However, war financing also had the effect of building up the "welfare state" and creating most of the institutions we know today (see The great leveller)
- This implied that taxes and public debt increased to unprecedented levels
- Maybe there is no growth model that makes sense without considering some type of public intervention

This lecture

- There are zillions of different models we could think about for addressing the role of government on growth (we already saw quite a few):
- **In this lecture we address the consequences of government financing, through debt and taxes, on the rate of growth**
- We assume that government expenditures generate an externality in production, which could be seen as investment in infrastructures (of a flow nature). However, the level of expenditures and the type of financing have an effect on the growth rate.
- The question is: **under which circumstances the need to finance expenditures will not jeopardize the positive effect of the externality, in the long run ?**

Assumptions

- Production of goods involves an **externality** associated to the services provided by the state;
- Services provided by the public sector are a **function of the government expenditures**;
- The government has to finance expenditures by **taxes and/or debt**, constrained by the government budget constraint (GBC)
- We assume that the government uses a **rule** of keeping the debt over the GDP ratio (B/Y) constant

Workings of the model

- There are two extreme **financing strategies**: tax finance (debt ratio equal to zero), debt finance (tax rate equal to zero)
- Taxes will distort the incentives to capital accumulation: this implies the model has externalities that are not internalized
- If the GBC does not need to be permanently balanced, this changes the rate of growth of the economy
- However, if there is a cap on government borrowing there is a **feedback mechanism** requiring the increase in taxes

Conclusions

- Government expenditures have a **positive** effect on the growth rate
- However, the **type of financing matters** for the long run level of the ratio G/Y:
  - tax financing has an ambiguous effect on the growth rate (it increases the long run interest before taxes but the effect on the net rate is ambiguous)
  - debt financing, such that B/Y is constant, can have a negative effect on growth because the implied sustainability requires that the long-run interest rate should be reduced

The model

Private sector

- Chooses $(C(t))_{t\geq 0}$ to maximize

$$\int_0^{\infty} \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt$$

  C is consumption

- subject to the budget constraint

$$\dot{K} + \dot{B} = (1-\tau)\left(r(t)B(t) + Y(t)\right) - C(t)$$

  B government bonds, K private capital, τ tax rate

- non-Ponzi game condition

$$\lim_{t\to\infty} \left(K(t) + B(t)\right) e^{\int_0^t r(s)\, ds} \geq 0.$$

- I am assuming that there is only an income tax with a flat rate (if K includes human capital a labor tax is implicitly assumed along with capital tax)

The model

Private sector: first order conditions

- Euler equation
$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(t)-\rho}{\theta} \tag{1}$$
- Budget constraint
$$\dot{K} + \dot{B} = (1-\tau)\left(r(t)B(t) + Y(t)\right) - C(t) \tag{2}$$
- Transversality condition
$$\lim_{t\to\infty} C(t)^{-\theta}(K(t)+B(t))e^{-\rho t} = 0.$$

The model

Production and arbitrage in financial markets

- Production technology

$$Y(t) = K(t)^{\alpha} G(t)^{1-\alpha},\ 0 < \alpha < 1 \tag{3}$$

  G government expendititures

- arbitrage condition between capital and government debt

$$r(t) = \alpha K(t)^{\alpha-1} G(t)^{1-\alpha}. \tag{4}$$

The model

The government

- GBC: government budget constraint

$$\dot{B} = \underbrace{r(t)B(t)}_{\text{interest payments}} + \underbrace{G(t) - \tau\left(r(t)B(t) + Y(t)\right)}_{\text{primary deficit}} \tag{5}$$

- Policy rule: the government keeps the ratio of debt over GDP constant

$$B(t) = \bar{b}\,Y(t) \tag{6}$$

where $\bar{b}$ is a number (Maastricht rule 60%)

The aggregate constraint

- Consolidating the private and government constraints yields the aggregate constraint of the economy

$$\dot{K} = Y(t) - C(t) - G(t)$$

which turns out to be equal to the equilibrium in the goods' market

$$Y(t) = C(t) + I(t) + G(t)$$

where $I = \dot{K}$

The DGE

Is represented by fur equations:

- The Euler equation

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(K,G) - \rho}{\theta}$$

- the aggregate budget constraint

$$\dot{K} = Y(K,G) - C - G$$

- the GBC

$$\dot{B} = r(K,G)B + G - \tau\left(r(K,G)B + Y(K,G)\right)$$

- the policy rule

$$B = \bar{b}Y$$

Transforming the DGE

- In order to reduce the dimensionality of the model, we define two relationships
- The ratio of government expenditures over the GDP

$$g \equiv \frac{G}{Y}$$

- The ratio of consumption over the capital stock

$$z \equiv \frac{C}{K}$$

The transformed DGE

The transformed Euler equation

- From $g \equiv \dfrac{G}{Y}$ then

$$Y = A(g)K, \text{ for } A(g) \equiv g^{\frac{1-\alpha}{\alpha}}, \; A' > 0$$

  Externality effect: an increase in g increases the TFP (A)

- Then the rate of growth of GDP is (taking log-derivatives)

$$\gamma_y = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g}$$

- The real interest function is a positive function of g

$$\boxed{r(g) = \alpha A(g),}$$

  $r' > 0$ because $A' > 0$

The transformed DGE

The transformed aggregate constraint

- Then the Euler equation becomes

$$\frac{\dot{C}}{C} = \gamma(g) \equiv \frac{(1-\tau)r(g) - \rho}{\theta}$$

  where $\gamma(g)$ is the rate of growth of consumption

- The rate of growth for private capital is

$$\frac{\dot{K}}{K} = \gamma_k(g, z) \equiv (1-g)A(g) - z$$

  where $\gamma_k(g)$ is the rate of growth of the capital stock, and $z \equiv C/K$

- Then

$$\frac{\dot{z}}{z} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K}$$

The transformed DGE

The transformed aggregate constraint

- Substituting

$$\frac{\dot{z}}{z} = \gamma_z(g,z) = \gamma(g) - \gamma_k(g,z) = z - z(g)$$

  where

$$z(g) \equiv \frac{(\theta(1-g) - \alpha(1-\tau))\, A(g) + \rho}{\theta}$$

  can be rationalized as a **Laffer curve**: it includes two effects:

  - a **positive** net effect of g on GDP (bigger for small values of g): $(1-g)A(g)$
  - a **negative** net effect of taxes over r which decreases the rate of growth: $(1-\tau)r(g) = (1-\tau)\alpha A(g)$

The transformed DGE

The dynamics of b

- Defining the government debt over GDP ratio by $b = \frac{B}{Y}$ we get

$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{Y}}{Y} = \gamma_b - \gamma_y$$

- where

$$\frac{\dot{B}}{B} = \gamma_b(g) = (1-\tau)r(g) + \frac{G - \tau Y}{B} = (1-\tau)r(g) + \frac{g-\tau}{b}$$

- Then, the dynamics of public debt over GDP is driven by

$$\frac{\dot{b}}{b} = (1-\tau)r(g) + \frac{g-\tau}{b} - \frac{\dot{K}}{K} - \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g} \tag{7}$$

The transformed DGE

The dynamics of g

- **Assumption**: introducing the policy rule $b = \bar{b}$ then $\dot{b} = 0$
- Then, we solve equation (7) to $\dot{g}/g$

$$\frac{\dot{g}}{g} = \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - \frac{\dot{K}}{K}\right) =$$
$$= \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - ((1-g)A(g) - z)\right) =$$
$$= \frac{\alpha}{1-\alpha}\left(z - \zeta(g)\right)$$

where

$$\zeta(g) \equiv \left((1-g) - \alpha(1-\tau)\right) A(g) + \frac{\tau - g}{\bar{b}}.$$

The DGE in detrended variables

- The DGE in detrended variables is

$$\begin{cases} \dot{g} = \frac{\alpha}{1-\alpha}\left(z - \zeta(g;\tau,\bar{b})\right)\ g \\ \dot{z} = \left(z - z(g)\right)\ z \end{cases} \tag{8}$$

where the long run C/K ratio for a constant g

$$\zeta(g;\tau,\bar{b}) \equiv \left((1-g) - \alpha(1-\tau)\right) A(g) + \frac{\tau - g}{\bar{b}}.$$

and the Laffer-curve is

$$z(g) \equiv \frac{\left(\theta(1-g) - \alpha(1-\tau)\right) A(g) + \rho}{\theta}$$

- for $g \in (0,1)$ and $z > 0$

The long run level of g

- In system (8) we set $\dot{z} = \dot{g} = 0$.
- The long run level of g is a function of τ and $\bar{b}$:

$$\bar{g} = \bar{g}(\tau, \bar{b}) = \{g : \ \Phi(g) \equiv z(g) - \zeta(g; \tau, \bar{b}) = 0\}$$

  where

$$\Phi(g, \tau, \bar{b}) \equiv \frac{(1-\tau)(\theta-1)}{\theta} r(g) + \frac{g-\tau}{\bar{b}} + \frac{\rho}{\theta}$$

- The steady state satisfies

$$\bar{g} \le \tau \le \frac{\rho}{\theta}\bar{b}$$

  only if $\tau > \frac{\rho}{\theta}\bar{b}$
- Next we prove that $\bar{g}_\tau > 0$ and $\bar{g}_{\bar{b}} < 0$

The long run level of g

- We assume that ($0 < \tau < 1$ and $\theta > 1$)

$$\Psi \equiv \frac{\partial \Phi(g,.)}{\partial g}\Big|_{g=\bar{g}} = \frac{(1-\tau)(\theta-1)r^{'}(\bar{g})}{\theta} + \frac{1}{\bar{\bar{b}}} > 0$$

- The partial derivatives of $\bar{g}$ (evaluated at the steady state are):
  - the effect of an increase in τ is positive (increases in the tax rate can finance increase in g)

$$\frac{\partial \bar{g}}{\partial \tau}\Big|_{g=\bar{g}} = \frac{(1-\tau)r^{'}(\bar{g}) - r(\bar{g})}{\theta\, \bar{b}^2\, \Psi} > 0$$

    if $(1-\tau)(1-\alpha) - \alpha\bar{g} > 0$;
  - the effect of increases of $\bar{b}$ on $\bar{g}$ is negative

$$\frac{\partial \bar{g}}{\partial \bar{\bar{b}}}\Big|_{g=\bar{g}} = \frac{(\bar{g}-\tau)\theta}{\theta\, \bar{b}^2\, \Psi} < 0$$

    because $\Phi(g,\tau) = 0$ only if $\bar{g} < \tau$; financing a higher level of debt competes with public expenditure

The function $\Phi(g)$

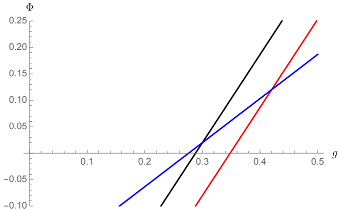

Figure: Function $\Phi(g, \tau, \bar{b})$ as a function of g. Increase in τ (red), increase in $\bar{b}$ (in blue)

Long-run interest rate

- The long run interest rate is

$$\bar{r} = r(\bar{g}) = R(\tau, \bar{b})$$

- As $r'(g) > 0$, then
  - an increase in the tax rate τ increases the long-run interest rate (through the externality of public expenditures channel): $\partial R / \partial \tau > 0$
  - an increase in the level of government debt $\bar{b}$ reduces the long run rate of return of capital (crowding out effect): $\partial R / \partial \bar{b} < 0$

Long-run growth rate

- The long run growth rate is a positive function of the steady state g ($\bar{g}$):

$$\boxed{\bar{\gamma} = \gamma(\bar{g}) = \frac{(1-\tau)r(\bar{g}) - \rho}{\theta}}$$

- then the long-run growth rate depends on the public financing policy

$$\boxed{\bar{\gamma} = \gamma(\tau, \bar{b})}$$

- We can prove that γ_τ is ambiguous and $\gamma_{\bar{b}} < 0$

Long-run growth rate

Policy instruments and the growth rate

- The effect of the tax rate in the rate of economic growth

$$\frac{\partial \gamma}{\partial \tau} = -\frac{r(\bar{g})}{\theta} + \frac{(1-\tau)}{\theta} r^{'}(\bar{g}) \frac{\partial \bar{g}}{\partial \tau}$$

  is **ambiguous** because:
  - there is a negative direct effect (incentive effect on the reduction of savings)
  - and a positive indirect effect through financing the externality effect of g ion r (under the assumption if $(1-\tau)(1-\alpha) > \alpha\bar{g}$)
- depending on the level of τ one of the two effects can prevail: the first effect is stronger for very high of very low tax rates
- there is a tax rate that maximizes the rate of growth

Effect of the tax rate on the long run growth rate

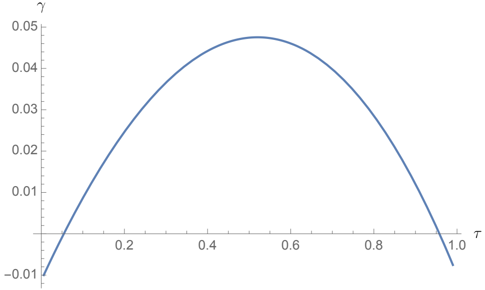

Figure: A Laffer curve (sort of): value of γ for different levels of the tax rate τ

Long-run growth rate

Policy instruments and the growth rate

- The effect of the debt ratio in the rate of economic growth

$$\frac{\partial \gamma}{\partial \bar{\bar{b}}} = \frac{(1-\tau)}{\theta} r^{'}(\bar{g}) \frac{\partial \bar{g}}{\partial \bar{\bar{b}}} < 0$$

  the effect on the growth rate is negative

- as a result of the crowding out effect: $\frac{R}{\partial \bar{\bar{b}}} < 0$

Transitional dynamics

- We can study the local dynamics of the model by finding the Jacobian of system (8) and evaluating it at the steady state $(\bar{g}, \bar{z})$ where $\bar{z} = z(\bar{g}) = \zeta(\bar{g})$
- the Jacobian is

$$J(g, z) = \begin{pmatrix} \frac{\alpha}{1-\alpha}\left(z - \zeta(g) - g\,\zeta^{'}(g)\right) & \frac{\alpha}{1-\alpha} g \\ -z\,z^{'}(g) & 2z - z(g) \end{pmatrix}$$

- evaluating at the steady state we have

$$J(\bar{g}, \bar{z}) = \begin{pmatrix} -\frac{\alpha}{1-\alpha}\bar{g}\,\zeta^{'}(\bar{g}) & \frac{\alpha}{1-\alpha}\bar{g} \\ -\bar{z}\,z^{'}(\bar{g}) & \bar{z} \end{pmatrix}$$

- it has determinant

$$\det\left(J(\bar{g}, \bar{z})\right) = \frac{\alpha}{1-\alpha}\,\bar{g}\,\bar{z}\left(z^{'}(\bar{g}) - \zeta^{'}(\bar{g})\right) = \frac{\alpha}{1-\alpha}\,\bar{g}\,\bar{z}\,\Psi > 0$$

Transitional dynamics

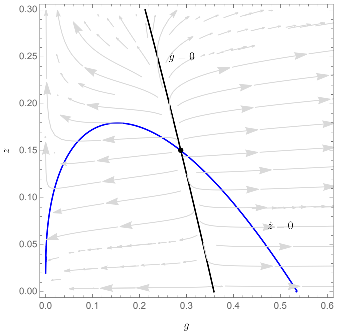

Figure: For $\theta = 1$, $\alpha = 0.7$, $\rho = 0.02$, $\tau = 0.3$ and $\bar{b} = 0.6$. The steady state is unstable: there is no transitional dynamics for $\Psi > 0$

Growth facts

Effects of policy parameters τ and $\bar{b}$

- Long-run growth rate:
  - although a higher g increases the long run growth rate
  - the need to finance it generates a countervailing effect
    Conclusion: while debt financing reduces the long run growth rate, tax finance may reduce or increase the long run growth rate, although the positive effect dominates for intermediate levels of the tax rate.
- There is no transitional dynamics for $\Psi > 0$. The only dynamics is long-run dynamics. This is a consequence of setting B/Y constant.

A different rule

- Now assume that there is a rule on the government deficit

$$\frac{\dot{B}}{Y} \leq \beta \, \text{EU level:3\%}$$

- To simplify assume that it is followed strictly $\frac{\dot{B}}{Y} = \beta$
- Show that now the dynamic system becomes

$$\dot{g} = \big((1-g)\, A(g) - z \big) \, (\beta + \tau - g)$$
$$\dot{z} = \big(z - z(g) \big) \, z$$

only the first equation changes

A different rule: conclusions

- Now the growth rate is

$$\bar{\gamma} = \frac{(1-\tau)\, r(\bar{g}) - \rho}{\theta}$$

  where $\bar{g} = \beta + \tau$
- The function $\gamma(\tau)$ has the same shape
- If β is higher the growth rate is higher as well
- There is transitional dynamics converging to the BGP
- But we have to check the transversality condition

Transitional dynamics

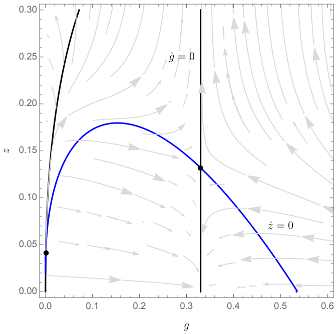

Figure: For $\theta = 1$, $\alpha = 0.7$, $\rho = 0.02$, $\tau = 0.3$ and $\beta = 0.03$. Now there are two steady states. There is transitional dynamics associated to the "high" growth steady state

References

- Barro (1990)

R. Barro. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98: S103–S125, 1990.