Growth and natural resources

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Natural capital and the GDP distribution

Data for 1980 and 2017

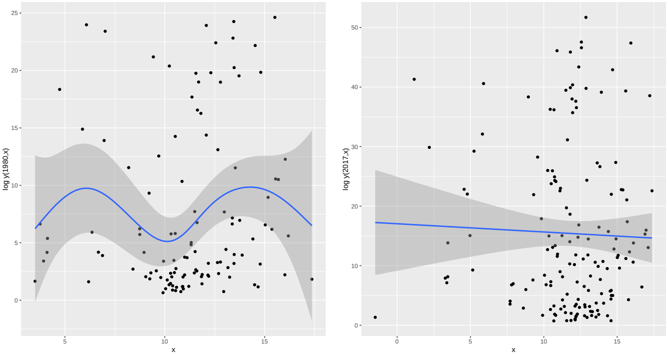


Figure: Sources: Feenstra et al. (2015) (PWT 10 series "rgdpo") and World Bank "total natural resource rents as a proportion of the GDP" (https://data.worldbank.org/indicator/NY.GDP.TOTL.RT.ZS)

There is not a systematic relationship between gdp per capita and the endowment of natural resources

What is natural capital

- It includes several dimensions: biodiversity see Dasgupta review, physical environment, climate, etc
- For a systematic measurement see World Bank

Types of natural capital

From the economic point of view, the major distinction is between **renewable and non-renewable resources**:

- A **non-renewable resource** has a similar role as land in the Malthusian model: (1) it is non-reproducible; (2) the total amount is limited; (3) it tends to make the economy to converge to a steady state, implying **no long-run growth**
- A **renewable resources**: (1) has have a reproduction mechanism; (2) the total amount is practically unlimited; (3) it tends to allow for **long-run growth**

Growth and the environment

From the perspective of modelling

1. Natural capital effects in the economy
   - The dynamics of natural capital can ultimately determine the dynamics of growth
   - Natural capital has an amenity value (effect on the utility of consumption)
   - Natural capital provides an input for production
   - It can also be a source of output
2. Effects of the economy in the environment
   - Negative: pollution, depletion of natural resources, climate change
   - Positive: cleaning, decarbonising, natural preservation, abatment policies
3. Technical progress: de-materialization may be the key to the existence of both growth and sustainability

Environment in growth models

- There is a reference model for studying climate change: DICE model of William Nordhaus and DICE model by the author
- The core model is

$$\max_c \int_0^\infty u(C(t))\, e^{-\rho t} dt$$

$$\text{subject to}$$

$$C(t) = f(E(t)) - h(M(t))$$

$$\dot{M} = \beta\, E(t) - \delta\, M(t)$$

  where E emissions, M stock of carbon.
- Trade-off between consumption and the stock of carbon: production means emissions, $f(E)$, but is negatively impacted by the stock of carbon $h(M)$, but emissions increase the stock of carbon
- No growth here !

A simple model

Problem

- The main question: can we have both growth and no-depletion of natural capital ?
- Can technical progress (in the sense of de-materialization) allow for both ?

Assumptions

- the natural resource is (at least partly) renewable;
- the natural resource has an amenity value for the consumer;
- production uses a natural resource as the only input;
- but use in production depletes its stock;
- technical progress takes the form of de-materialization;

Conclusions

- the feasible growth rate is limited by the rate of technical progress and the sum of the rate of technical progress and the rate of regeneration of the natural resource
- then growth with positive growth rates is sustainable, although along a narrow corridor.

The structure of the economy

Product market

- production function

$$Y(t) = A(t)E(t)$$

  A is TFP and E are emissions (=resource depletion)

- (exogenous) technical progress

$$A(t) = A_0 e^{\gamma_a t}$$

- equilibrium in the product market

$$Y(t) = C(t)$$

- **then**

$$E(t) = \frac{C(t)}{A(0)} e^{-\gamma_a t}$$

  technical progress involves dematerialization

Natural resource dynamics

- natural resource accumulation equation

$$\dot{N} = \mu N(t) - E(t), \;\; \mu > 0$$

  where $N(0) = N_0$ is given
- **then**

$$\dot{N} = \mu N(t) - \frac{C(t)}{A(0)} e^{-\gamma_a t}$$

Consumers' preferences

- the instantaneous utility function is

$$u(C, N) = \frac{(CN^{\varphi})^{1-\sigma}}{1-\sigma}$$

- observtions: φ parameterizes the amenity services provided by natural resources; observe that the utility function is homogenous of degree $(1-\sigma)(1+\varphi)$

The problem

- The central planner problem (in variables with trend) is

$$\max_{C(\cdot)} \int_0^\infty \frac{\left(C(t)\, N(t)^{\varphi}\right)^{1-\sigma}}{1-\sigma}\, e^{-\rho t}\, dt,\;\; \rho > 0, \sigma > 0, \varphi > 0$$

subject to

$$\dot{N} = \mu N(t) - \frac{C(t)}{A(t)}$$

$$N(0) = n_0 \text{given}$$

- where $\dot{A} = \gamma_a\, A$ and $A(0) = A_0$ given

Detrending

- we assume that consumption and the stock of natural resources can be written as

$$C(t) = c(t)\,e^{\gamma t},\ N(t) = n(t)\,e^{\gamma_n t}$$

- **then** from the resource accumulation equation is

$$\dot{n} = (\mu - \gamma_n)\,n(t) - A_0^{-1}\,c(t)\,e^{(\gamma - \gamma_a - \gamma_n)\,t}$$

- to get the autonomous ODE

$$\dot{n} = (\mu - \gamma_n)\,n - A_0^{-1}\,c$$

we make $\gamma = \gamma_a + \gamma_n$

Detrending

- the instantaneous utility function is

$$u(C, N) = \frac{(CN^{\varphi})^{1-\sigma}}{1-\sigma}$$

- in detrended variables, we get

$$u(C, N) = e^{\gamma_u t} u(c, n) \equiv e^{\gamma_u t} \frac{(cn^{\varphi})^{1-\sigma}}{1-\sigma}$$

  where the rate of growth of utility is

$$\gamma_u = (1-\sigma)[\gamma_a + (1+\varphi)\gamma_n]$$

The problem in detrended variables

- Planner's problem written in detrended variables (c, n)

$$\max_{(c(t))_{t\in[0\infty)}} \int_0^{\infty} \frac{(c(t)n(t)^{\varphi})^{1-\sigma}}{1-\sigma} e^{-\rho^* t}$$

subject to

$$\dot{n} = (\mu - \gamma_n)n - A(0)^{-1}c$$

$$n(0) = N_0 \text{ given}$$

$$\lim_{t\to\infty} n(t)\, e^{\gamma_n t} \geq 0$$

- where $\rho^* \equiv \rho - \gamma_u = \rho - (1-\sigma)[\gamma_a + (1+\varphi)\gamma_n]$ and γ_n is unknown,
- **assumption**

$$(1-\sigma)(1+\varphi)\mu < \rho - (1-\sigma)\gamma_a < (1+\varphi)\mu \qquad \text{(A)}$$

- this guarantees sustainability and positive growth

Optimality conditions

- optimal consumption

$$A_0\, c(t)^{-\sigma} n(t)^{\varphi(1-\sigma)} = q(t) \tag{1}$$

- Euler equation

$$\dot{q} = q(t)(\rho^* - \mu + \gamma_n) - \varphi c(t)^{1-\sigma} n^{\varphi(1-\sigma)-1}$$

- Substituting (1) the MHDS is

$$\begin{cases} \dot{n} = (\mu - \gamma_n) n - A(0)^{-1} c \\ \dot{q} = q(t)\left(\rho^* - \mu + \gamma_n - \dfrac{\varphi}{A(0)} \dfrac{c}{n}\right) \\ \lim_{t\to\infty} q(t) n(t) e^{-\rho^* t} = 0 & \text{transversality condition} \\ n(0) = N_0 & \text{given} \end{cases}$$

The MHDS

- Taking log-derivative to (1) we get

$$-\sigma\frac{\dot{c}}{c}+\varphi(1-\sigma)\frac{\dot{n}}{n}=\frac{\dot{q}}{q}$$

- we can get the MHDS for (c, n)

$$\begin{aligned}\frac{\dot{c}}{c} &= \frac{\mu(1+\varphi(1-\sigma))+(1-\sigma)\gamma_a-\sigma\gamma_n-\rho}{\sigma}+\varphi\frac{c}{A(0)n}\\ \frac{\dot{n}}{n} &= \mu-\gamma_n-\frac{c}{A(0)n}\end{aligned}$$

Long-run rate of growth

- solving $\frac{\dot{c}}{c} = 0$ and $\frac{\dot{n}}{n} = 0$ for γ_n and c/n, we get:
- the long-run rate of growth

$$\bar{\gamma}_n = \frac{(1+\varphi)\mu + (1-\sigma)\gamma_a - \rho}{\sigma(1+\varphi)}$$

- and the long-run consumption-resources ratio

$$\frac{\bar{c}}{\bar{n}} = A_0(\mu - \bar{\gamma}_n)$$

- as $\bar{y} = \bar{c}$ from the product market equilibrium condition then

$$\bar{y} = A_0(\mu - \bar{\gamma}_n)\bar{n}$$

Long-run rate of growth

- **Proposition**: if assumption (A) holds then

$$0 < \bar{\gamma}_n < \mu$$

- **Intuition: the (economic) rate of growth of the natural resource is limited by the natural reproduction rate but can still be positive.**
- Proof:
- $\bar{\gamma}_n > 0$ if and only if

$$(1+\varphi)\mu > \rho - (1-\sigma)\gamma_a$$

- $\bar{\gamma}_n < \mu$ if and only if

$$(1+\varphi)\mu + (1-\sigma)\gamma_a < \rho + \mu\sigma(1+\varphi)$$

  which is equivalent to

$$\rho - (1-\sigma)\gamma_a > (1-\sigma)(1+\varphi)\mu$$

Transitional dynamics

- Defining $z(t) \equiv A_0 \dfrac{c(t)}{n(t)}$ and substituting $\gamma_n = \bar{\gamma}_n$ into the MHDS we get
$$\frac{\dot{z}}{z} = (1+\varphi)(z - \bar{z})$$
where $\bar{z} = \mu - \bar{\gamma}_n$
- as the equation is unstable, the transversality condition only holds if $z(t) = \bar{z}$ for $t \in [0, \infty)$
- as $\bar{z} = z(0)$ we set
$$\bar{c} = \bar{y} = A_0(\mu - \gamma_n)N_0$$

Growth facts

1. The long run growth rate is

$$\bar{\gamma} = \gamma_a + \bar{\gamma}_n$$

Intuition: if the assumption on the parameters holds then the growth rate is limited by the natural renewal rate and the growth in dematerialization

$$\gamma_a < \gamma < \gamma_a + \mu$$

2. the long run GDP level is a positive function of the initial stock of natural resources (counterfactual)

$$\bar{y} = \bar{c} = A_0(\mu - \bar{\gamma}_n)N_0 = A_0 N_0 \frac{(\sigma - 1)((1+\varphi)\mu + \gamma_a) + \rho}{\sigma(1+\varphi)}$$

3. there is no transitional dynamics (counterfactual)

Comment

- Looking at the initial figure 1, and accepting the measurement of the stock of the natural resources, it does not look like there is a systematic effect between the level of the GDP per capita and the stock of natural resources (but the model gives a **conditional** and the data provides an **unconditional** relationship)
- Do these results generalize to the introduction of capital accumulation ?
- That is: does introducing capital accumulation alleviates the ”natural” constraint ?
- As we can see next, the answer is no !

Generalization

A model with capital

$$\max_{([C(t),E(t)]_{t=0}^{\infty})} \int_0^{\infty} \frac{1}{1-\sigma} \left(C(t) N(t)^{\varphi} \right)^{1-\sigma} e^{-\rho t} dt, \quad \rho > 0, \ \sigma > 0, \ \varphi > 0$$

subject to

$$\dot{K} = A(t) K(t)^{\alpha} E(t)^{1-\sigma} - C(t) - \delta K(t), \ 0 < \alpha < 1$$

$$\dot{N} = \mu N(t) - E(t), \ \mu > 0$$

A model with capital

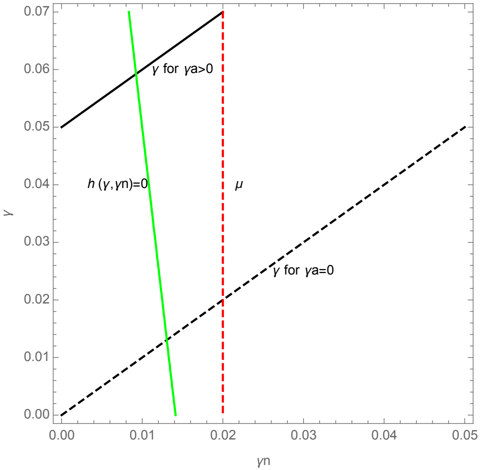
- We find that the long-run growth rate is now

$$\gamma = \frac{1}{1-\alpha}\gamma_a + \gamma_n$$

- then $\gamma \geq \gamma_n \iff \gamma_a \geq 0$
- again $\gamma_n < \mu$ and γ_n is determined implicitly from

$$h(\gamma,\gamma_n) \equiv \varphi\,(\mu-\gamma_n)\,\big(\rho+(\sigma-1)(\gamma+\varphi\,\gamma_n)+(1-\alpha)(\delta+\gamma)\big) = 0$$

Growth, environmental and technical progress



- An increase in productivity in production: decreases the rate of growth γ_n but increases γ
- There is still sustainability because $0 < \gamma_n < \mu$

References

- On William Nordhaus contributions
- (Aghion and Howitt, 2009, ch. 16)

Philippe Aghion and Peter Howitt. *The Economics of Growth*. MIT Press, 2009.

Robert C. Feenstra, Robert Inklaar, and Marcel P. Timmer. The Next Generation of the Penn World Table. *American Economic Review*, 105(10):3150–3182, October 2015. URL https://ideas.repec.org/a/aea/aecrev/v105y2015i10p3150-82.html.