Economic Growth Theory:

Problem set 10:

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5.5.2021

Growth and the environment

1. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N - P(t)$, where μ is the renewal rate and P is the use of the resource in production. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods, which are used only in consumption. We assume a centralized economy in which the central planner maximises the utility function

$$\int_0^\infty \left(\ln\left(C(t)\right) + \varphi \ln\left(N(t)\right)\right) e^{-\rho t} dt$$

where the rate of time preference, ρ, and the utility weight associated by consumers to the environment, φ, are both positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} e^{-\rho t} > 0$.

   (a) Write the first order conditions for optimality.

   (b) Prove that the optimal level for the natural resource is $N(t) = N_0 e^{\gamma t}$ where $\gamma = \mu - A\rho/(A+\varphi)$.

   (c) What implications on the growth facts can we draw from this model ?

2. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N - P(t)$, where μ is the renewal rate and P is the use of the resource in production. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods, which are used only in consumption. We assume a decentralised economy in which a consumer with weight $0 < \alpha < 1$ only consider her/his effect on total demand, $D(t) = C(t)^{\alpha}\mathbf{C}(t)^{1-\alpha}$ where C is the representative agent consumption and $\mathbf{C}$ is the aggregate consumption.

   The representative consumer maximises the utility function

$$\int_0^{\infty} \ln(C(t))e^{-\rho t}dt$$

   where the rate of time preference, ρ is positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} N(t)e^{-\rho t} > 0$.

   (a) Write the first order conditions for optimality for the representative consumer as a dynamic system in (C, N).

   (b) Write the dynamic system for the aggregate economy and solve it.

   (c) Discuss the implications for the growth facts that we can draw from this model ?

3. Let the dynamics of the endowment of natural resources be given by $\dot{N} = \mu N(t)^{\alpha}X(t)^{1-\alpha} - P(t)$, where μ is the renewal rate and P is the use of the resource in production, X is the expenditure in environmental preservation and $\alpha \in (0, 1)$. The production function is $Y(t) = AP(t)$ where A is constant and Y is the output of final goods. The final good is used in consumption and environmental preservation, such that the equilibrium condition $Y = C + X$ holds. We assume a centralized economy in which the central planner has the optimality criterium

$$\max_{C,X} \int_0^{\infty} \ln(C(t))e^{-\rho t}dt$$

   Â where the rate of time preference, ρ, is positive. The initial stock of natural resources is $N(0) = N_0$ given and assume the terminal constraint $\lim_{t\to\infty} e^{-\rho t} > 0$.

   (a) Write the first order conditions for optimality as a system in (Q, N), where Q is the co-state variable.

(b) Find the optimal solution for $N(t)$ (hint: reduce the dimensionality of the system by defining $V(t) = Q(t)N(t)$).

(c) What implications on the growth facts can we draw from this model ?