

Economic Growth Theory:

Problem set 5:

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Uzawa-Lucas models

1. Consider the following version of the Uzawa-Lucas model with the intertemporal utility functional

$$\max_{C, K_1, H_1, H_2} \int_0^{\infty} \ln C(t) e^{-\rho t} dt$$

where $\rho > 0$ is the rate of time preference. The problem has the following constraints accumulation equations for stocks of physical and human capital

$$\dot{K} = Y_1(t) - C(t)$$

$$\dot{H} = Y_2(t)$$

where Y_1 is the output of manufacturing and Y_2 is the output of education, and K and H are the aggregate stocks of physical and human capital, respectively, and the allocation constraints of the stocks between the two sectors satisfies

$$K(t) = K_1(t)$$

$$H(t) = H_1(t) + H_2(t)$$

where K_j is physical capital and H_j human capital allocated to sector $j = 1, 2$. Assume that the production functions for manufacturing and education are

$$Y_1(t) = A_1 K_1(t)^\alpha H_1(t)^{1-\alpha}$$

$$Y_2(t) = A_2 H_2(t)$$

where $\alpha \in (0, 1)$ is the share of capital in manufacturing. At last assume that $\rho + A_2(\theta - 1) > 0$ and that all the capital stocks are bounded in present-value terms when $t \rightarrow \infty$.

- (a) Respecify the model with detrended variables by assuming there is a balanced growth path
- (b) Find the optimality conditions by applying the Pontryagin's maximum principle
- (c) Find the real rates of return and the optimal allocation of physical and human capital among sectors. We find that the wage rate is constant. Why?
- (d) Find the long run levels for the growth rate, the relative prices of factors, and the long run levels for physical capital and human capital.
- (e) Write the MHDS in detrended variables and find a linear approximation in the neighborhood of the balanced growth path
- (f) Solve the linearized system
- (g) Draw the phase diagram for a projection in the (k, h) -space, assuming that k_0/h_0 is "too high".
- (h) Assume that the economy is at the BGP. Study the effect of a permanent increase in the productivity of the education sector A_2 , in the long-run growth rate and in the long run level of the GDP.
- (i) Assume that the economy is at the BGP. Study the effect of a permanent increase in the productivity of the manufacturing sector A_1 , in the long-run growth rate and in the long run level of the GDP.