Economic Growth Theory:

Problem set 8:

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Automation and growth

1Part 1 Consider a Solow economy in which the production technology uses machines, labor and robots. While machines are imperfectly substitutable with labor and robots, robots and labor are perfectly substitutable. This basically means that a machine (ex a truck) can be driven by a human or a robot. In particular let aggregate output, Y, be produced using the production function

$$Y(t) = K(t)^{\alpha}\Big(L(t) + M(t)\Big)^{1-\alpha}, \text{ with } 0 < \alpha < 1,$$

where K, L and M denote machines, labor and robots, respectively. There is full employment and population grows at a rate $n > 0$. The initial levels of machines, population and robots are all given and positive, v.g. $K(0) = k_0 > 0$, $L(0) = l_0 > 0$ and $M(0) = m_0 > 0$. Furthermore, assume: (H1) savings finance gross investment in machines, i.e.$S(t) = \dot{K} + \delta K(t)$, where $\delta > 0$ is the depreciation rate, and $S(t) = s\,Y(t)$, with $0 < s < 1$; and (H2) robots grow exogenously at the growth rate $n > 0$.

(a) Denoting the per-capita output by $y = Y/L$, obtain a differential equation displaying the dynamics of y.

(b) Solve the linearized equation for y in the neighborhood of a (positivelly valued) steady state.

(c) Is there long-run growth in this economy ? Charaterize the behavior of per capita income in this economy, and in particular the consequences from increasing m_0.

Part 2 In the previous economy, consider all the previous assumptions, except H1 and H2. Assume instead that savings finance not only gross investments in machines but also investments in robots. If the proportion of savings financing machines is denoted by ρ, such that $0 < \rho < 1$, we have $\rho\, S(t) = \dot{K} + \delta K(t)$ and $(1-\rho)\, S(t) = \dot{M} + \delta M(t)$.

(a) Let $k = K/L$ and $m = M/L$ be the intensities of machines and robots. Obtain a differential equation system for the joint dynamics of k and m.

(b) Assume that $(1-\rho)\, k_0 = \rho\, m_0$. Prove that linearizing the solution in the neighborhood of the steady state we obtain

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\begin{aligned}
k(t) &= \frac{\tilde{k}}{1-\tilde{m}} + \left(k_0 - \frac{\tilde{k}}{1-\tilde{m}}\right) e^{\gamma t},\ t \in [0,\infty)\\
m(t) &= \left(\frac{1-\rho}{\rho}\right) k(t),\ t \in [0,\infty)
\end{aligned}
$$

where $\tilde{k} \equiv \left(\frac{s\rho}{n+\delta}\right)^{\frac{1}{1-\alpha}}$, $\tilde{m} \equiv \left(\frac{1-\rho}{\rho}\right)\tilde{k}$ and $\gamma \equiv (1-\alpha)(n+\delta)(\tilde{m}-1)$.

(c) Assume $\tilde{m} > 1$. Charaterize the long-run behavior of per capita income in this economy. Discuss your result by comparing it with what you have obtained in Part 1 (c).