Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Economics
Crescimento Económico (Growth Economics)
2020-2021

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Exam: **Época Normal** (First exam)
11.6.2021 (18.00h-21.00h, AF24)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. [`6 points (3,3)`] Please answer **two** of the following three questions.
   (a) Some political parties and some governments are discussing the possibility of introducing a waiving to the patent protection of COVID 19 vacines. Discuss the pros and cons of that measure in the light of growth models with endogenous R&D. In your answer: (a) identify, justifying, the type of model that would be appropriate to discuss that problem; and (b) briefly discuss the causes and consequences of a patent protection in the context of the model you have chosen.
   (b) Recent statistical evidence for Portugal shows that wages for young graduates (*licenciados*), aged between 24 and 35 years, have declined by 16.7% between 2010 and 2019. In the same period, the proportion of population in that age interval that graduated increased from 25.5% to 37.4%. According to the directed technical change model, how can we explain this evolution ? Start your answer with a brief description of main ideas which that model formalizes.
   (c) Assume you were commissioned to build a model for the relationship between economic growth and global warming. Which assumptions you would consider to build such a model ? Which type of growth dynamics would you expect to derive ?
2. [`7 points (1.5,1.5,1.5,1.5,1)`] Consider a version of the Solow model, in which there are two types of labor, in strictly positive quantities: skilled L_s, and unskilled labor L_u. The technology of production is represented by the production function

$$Y(t) = (K(t) + A\, L_s(t))^{\alpha} L_u(t)^{1-\alpha}$$

   where $0 < \alpha < 1$ and $A > 1$ measures the specific productivity of skilled labor.
   (a) Find the characteristics of the technology, implicit in the production function, as regards: (a) the gross complementarity/substitutability between the three factors; (b) the Inada properties ; and (c) the returns to scale.
   (b) From now on assume the following: first, the total population, L, grows at a constant rate $n > 0$; second, the distribution of population is constant, where the proportion of skilled population is $\ell = L_s/L$, for $0 < \ell < 1$; third, the savings function is $S(t) = sY(t)$, with $0 < s < 1$; and, at last, there is no depreciation of capital. Define the equilibrium for this economy and derive the accumulation equation for the detrended capital stock $k(t) \equiv K(t)/L(t)$.
   (c) Prove there is a unique long run level for k. Is uniqueness related to the Inada properties of the production function, as a function of k ? Justify.
   (d) Write the linearized dynamic equation in a neighborhood of the steady state. Solve it. Which type of growth dynamics is generated by this model ? Justify.
   (e) Assume there is a permanent increase in the proportion of skilled labour ℓ. Determine the effects on the long-run growth rate and the level of per capita output. Under which conditions there is a positive effect over the long-run level of output ? Provide an interpretation for your result.

3. [`7 points (1.5,1.5,1.5,1.5,1)`] Consider a centralized economy model in which the central planner's problem is

$$\max_{(C(t))_{t\geq 0}} \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta}\, L(t)\, e^{-\rho t} dt,$$

subject to the restriction $\dot{\mathbf{K}} = A\mathbf{K}(t) - C(t)L(t) - \delta\mathbf{K}(t)$, given $\mathbf{K}(0) = K_0 > 0$ and $\lim_{t\to\infty} e^{-At}\mathbf{K}(t) \geq 0$, where $\mathbf{K}(t) \equiv K(t)L(t)$ is the aggregate capital stock and $L(t)$ is total population, $C(t)$ is per-capita consumption level, and $K(t)$ is the per-capita capital stock. We assume that population grows exogenously as $L(t) = e^{nt}$, where n is the growth rate of the population. Consider the following assumptions over the parameters: $\theta > 0$ and $0 < \rho < (\theta - 1)(A - \delta) - \theta\, n$ where A is the TFP and δ is the depreciation rate of capital.

(a) Write the central planners's problem in terms of per-capita variables.

(b) Determine the optimality conditions as an initial-terminal value problem in the per-capita variables (C, K).

(c) Specify the model in (per-capita) detrended variables, and determine the long-run (endogenous) growth rate.

(d) Prove that the solution for the detrended variables is $k(t) = K_0$ and $c(t) = \beta K_0$, where $\beta \equiv \dfrac{(\theta-1)(A-\delta) - \theta n + \rho}{\theta}$.

(e) Discuss the growth properties of the model. What are the implications of an increase in the growth rate of population n ?