A New-Keynesian growth model

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New-Keynesian macroeconomics

- Specifies the micro-foundations of macroeconomics (as Neoclassical (NC) models but differently from Keynesian (ISLM) models)
- But assumes there is imperfect competition (and not perfect competition as NC models)
- It is focused on the existence of nominal and real rigidities for studying the effects of macroeconomic policy (fiscal and monetary policies)
- Has been extended recently along two several directions:
  - economic growth: R&D (next two lectures)
  - heterogeneity (HANK): addressing financial frictions
  - international trade

This lecture

Presents a model which has both

- NK features (monopolistic competition):
  - the final good is produced from a continuum of intermediate goods which are imperfectly sutstitutable;
  - every intermediate good is produced by a monopolist;
- *AK* features:
  - the engine of growth is capital accumulation
  - the production function for every variety is linear
- But the equilibrium is not Pareto optimal

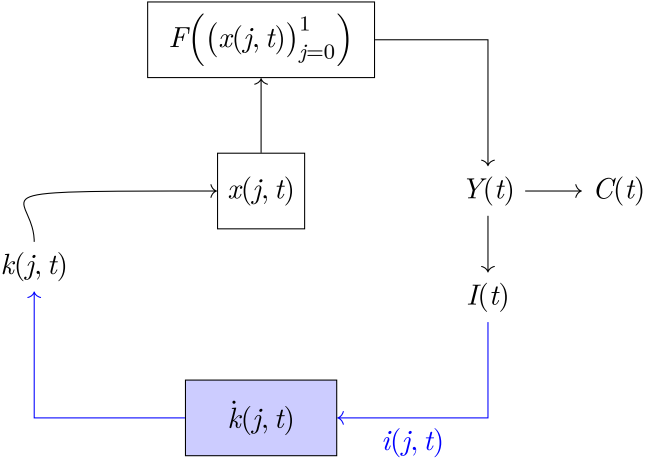
Results: implication for growth

- There is long-run growth and there is no transition dynamics
- The rate of growth depends negatively on the index Lerner (i.e, on the degree of imperfect competition)

Economic structure

- Environment:
  - there are two sectors: a competitive final good sector and a continuum of monopolistically competitive intermediate goods sectors
  - there is no entry and no exit
  - there is capital accumulation
- Technology:
  - final good production uses a continuum of intermediate goods
  - the final good is the only reproducible input
  - the dynamics of output is driven by the demand for capital by the intermediate good sector

The mechanics of the model



The structure of the model

- 1. Consumer problem (CP)
- 2. Final producer problem (FGPP)
- 3. Producers of intermediate goods (incumbents and entrants) (IGPP)
- 4. Aggregation, balance sheet and market clearing conditions
- 5. DGE model (DGE)

1. The household problem

- Households own firms (only intermediate good firms use capital)
- Earns capital income, consumes a final product and saves
- The problem

$$\max_{(C(t))_{t\in[0,\infty)}} \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \;\; \theta > 0$$

subject to

$$\dot{W} = r_w(t)\, W(t) - C(t) \tag{CP}$$

$W(0) = W_0$, initial condition

$\lim_{t\to\infty} e^{-R_w(t)} \, W(t) \geq 0$ NPG condition

where $R_w(t) = e^{\int_0^t r_w(s)\, ds}$ is the market return of capital

1. The consumer problem

- The first order conditions are

$$\dot{C} = \frac{C}{\theta}\,(r_w(t) - \rho) \tag{1}$$

$$\dot{W} = r_w(t)\,W(t) - C(t) \tag{2}$$

$$W(0) = W_0 \tag{3}$$

$$\lim_{t\to\infty} K(t)\,C(t)^{-\theta}\,e^{-\rho\,t} = 0 \tag{4}$$

2. Producer of the final good

- Buys intermediate goods and sells a final good
- Production function: Dixit and Stiglitz (1977)

$$Y(t) = F\big(\mathbf{x}(t)\big) = \Big(\int_0^1 x(j,t)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\Big)^{\frac{\varepsilon}{\varepsilon-1}}$$

$\varepsilon > 1$ elasticity of substitution between varieties

- $\mathbf{x}(t) \equiv \big(x(j,t)\big)_{j\in[0,1}$ intermediate inputs, non-storable

2. Producers of the final good (cont)

- Producer profit: sales minus total cost (nominal)

$$\pi^p(t) = P(t)\,Y(t) - \int_0^1 P(j,t)\,x(j,t)\,dj$$

$P(j,t)$ price of the intermediate good j

- The problem:

$$\max_{(x(j,t))_{j\in[0,1]}} \pi^p(t) \text{ such that } F\big(\mathbf{x}(t)\big) = Y(t) \qquad \text{(FGPP)}$$

- Obs: it is a price taker in all markets (only chooses quantities)

2. Producers of the final good (cont)

First order conditions:

- demand for intermediate good

$$x^d(j,t) = \left(\frac{P(t)}{P(j,t)}\right)^{\varepsilon} \; Y(t),\; j \in [0,1] \tag{5}$$

- where

$$P(t) = P^*(t) = \left(\int_0^1 P(j,t)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$$

3. Producer of intermediate good j

- There is one producer per variety j (it is a monopolist)
- This means that it takes the demand function for the final producer $x^d(j, t)$ as given
- Assumptions:
  - the production is a linear function of the capital stock $x(j, t) = A\, k(j, t)$
  - the technology is symmetric (i.e., homogeneous across varieties)
- Capital is produced by the final good sector.

3. Producer of intermediate good j

- The profit for producer j in real terms, deflated by the price of the final good

$$\pi(j,t) = \frac{P(j,t)\,x(j,t)}{P(t)}$$

$$\text{(using the demand function (5))}$$

$$= \left(\frac{x(j,t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}} x(j,t)$$

$$= x(j,t)^{1-\mu}\, Y(t)^{\mu}$$

- where

$$\mu \equiv \frac{1}{\varepsilon} \in (0,1)$$

is the Lerner index: higher μ means higher market power

3. Producer of intermediate good j

- The production function is

$$x(j,t) = A\,k(j,t)$$

- Therefore, the profit is

$$\pi(j,t) = \big(A\,k(j,t)\big)^{1-\mu}\;Y(t)^{\mu} \tag{6}$$

- depends on the own-capital of firm j and on the aggregate output

3. Producer of intermediate good j (cont.)

- The problem:

$$\max_{i(j,\cdot)} \int_0^\infty \big(\pi(j,t) - i(j,t)\big)\, e^{-R(t)}\, dt$$
$$\text{subject to}$$
$$\dot{k}(j,t) = i(j,t) - \delta\, k(j,t)$$
$$k(j,0) = k_0(j) \tag{IGPP}$$

- where $i(j,t)$ is gross investment (expenditure)
- and $\pi(j,t) - i(j,t)$ is the cash-flow in real terms

3. Producer of intermediate good j (cont.)

- first order condition

$$k(j,t) = \left(\frac{1-\mu}{r(t)+\delta}\right)\pi(j,t)$$

- Substitution in equation (15) yields

$$k^*(j,t) = k^*(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1}{\mu}}\frac{Y(t)}{A}$$

- Therefore, the optimal supply of variety j is

$$x^*(j,t) = x^*(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1}{\mu}} Y(t)$$

- Investment $i^*(j,t)$ is undetermined (it is determined residually at the equilibrium level)

General equilibrium

Recall:

- The consumer solves (CP)
- The producer of final goods solves (FGPP)
- The intermediate producers solve problems (IGPP)
- Aggregate accounting consistency condition
- Market equilibrium

4. Aggregation and accounting consistency

- Aggregate output of the final good

$$
\begin{aligned}
Y(t) &= \Big(\int_0^1 x^*(j,t)^{1-\mu} \, dj \Big)^{\frac{1}{1-\mu}} \\
&= \Big(\int_0^1 x^*(t)^{1-\mu} \, dj \Big)^{\frac{1}{1-\mu}} = x^*(t) \\
&= \Big(\frac{(1-\mu)\, A}{r(t)+\delta} \Big)^{\frac{1}{\mu}} \; Y(t)
\end{aligned}
$$

- is satisfied if and only if

$$
r(t) + \delta = (1-\mu)\, A
$$

- But we also have

$$
\begin{aligned}
Y(t) &= \Big(\int_0^1 x^*(j,t)^{1-\mu} \, dj \Big)^{\frac{1}{1-\mu}} \\
&= \Big(\int_0^1 A\, k^*(t)^{1-\mu} \, dj \Big)^{\frac{1}{1-\mu}} \\
&= A\, k^*(t)
\end{aligned}
$$

4. Aggregation and accounting consistency (cont)

- Aggregate capital stock

$$K(t) = \left(\int_0^1 k^*(j,t)^{1-\mu} dj \right)^{\frac{1}{1-\mu}} = k^*(t)$$

- Then

$$Y(t) = A\, K(t)$$

- Aggregate investment

$$I(t) = \left(\int_0^1 i^*(j,t)^{1-\mu} dj \right)^{\frac{1}{1-\mu}}$$

still to determine

4. Aggregation and accounting consistency (cont)

- Consistency condition: households own firms

$$W(t) = K(t)$$

- returns are equalized

$$r_w(t) = r(t) = (1-\mu)\,A - \delta$$

- Therefore the balance constraint (2) becomes

$$\dot{K} = \big((1-\mu)\,A - \delta\big)\,K - C \tag{7}$$

5. Market equilibrium

- Final good market equilibrium

$$Y(t) = C(t) + I(t)$$

- From $Y = AK$ and equation (7)

$$I(t) = \dot{K}(t) + (\mu A + \delta) K(t)$$

obs: imperfect competition reduces net investment $(\dot{K})$

- Furthermore, from $x^d(j,t) = x^*(j,t)$ yields

$$P(j,t) = P(t) \text{ for all } j \in [0,1]$$

General equilibrium

- The equilibrium is characterized by the system

$$\dot{K} = \big((1-\mu)\,A - \delta\big)\,K - C$$
$$\dot{C} = C\,\frac{(1-\mu)\,A - \delta - \rho}{\theta}$$
$$K(0) = k_0$$
$$\lim_{t\to\infty} K(t)\,C(t)^{-\theta}\,e^{-\rho\,t} = 0$$

- Recall that the GDP per capita is

$$Y(t) = A\,K(t)$$

The BGP

- Decomposing the variables: in the transition and trend components

$$K(t) = k_d(t)\, e^{\gamma\, t},\;\; C(t) = c_d(t)\, e^{\gamma\, t}$$

- yields the equilibrium in transition variables

$$\dot{k}_d = \big((1-\mu)\, A - \delta - \gamma\big)\, k_d - c_d$$

$$\dot{c}_d = c_d \left(\frac{(1-\mu)\, A - \delta - \rho}{\theta} - \gamma\right)$$

$$k_d(0) = k_0$$

$$\lim_{t\to\infty} k_d(t)\, c_d(t)^{-\theta}\, e^{-\beta\, t} = 0$$

where $\beta = (\theta - 1)\,\gamma + \rho$

The BGP

The steady state of this system allows for obtaining

- The endogenous **long-run growth rate**

$$\bar{\gamma} = \frac{(1-\mu)\,A - \delta - \rho}{\theta}$$

- the **level** variables, evaluated at $k_d = k_0$,

$$c_d = \bar{\beta}\,k_d = \bar{\beta}\,k_0$$

- where

$$\bar{\beta} = \frac{\big((1-\mu)\,A - \delta\big)\,(\theta - 1) + \rho}{\theta} > 0$$

for the verification of the transversality condition.

The BGP

- The equilibrium displays a BGP

$$\bar{K}(t) = k_0 \, e^{\bar{\gamma}\, t}, \; t \in [0, \infty)$$
$$\bar{C}(t) = \bar{\beta} \, k_0 \, e^{\bar{\gamma}\, t}, \; t \in [0, \infty)$$

- The GDP along the BGP is

$$\bar{Y}(t) = y_d \, e^{\bar{\gamma}\, t}, \; t \in [0, \infty)$$

where $y_d = A \, k_0$

Growth facts

- This model displays **long-run growth** but **no transitional dynamics**
- The **endogenous long-run growth rate** is

$$\bar{\gamma} = \Gamma(\overset{+}{A}, \overset{-}{\mu}) = \frac{(1-\mu)\,A - \delta - \rho}{\theta}$$

  increasing with productivity and the degree of competition (low μ or high ε)
- The level of the GDP is increasing with productivity $y_d = A\,k_0$

Growth facts

Comparison with an analogous AK model:

- the rate of growth is lower

$$\bar{\gamma} < \gamma_{AK} = \frac{A - \delta - \rho}{\theta}$$

- consumption is also lower

$$\bar{\beta} < \beta_{AK} = \frac{(A - \delta), \ (\theta - 1) + \rho}{\theta} > 0$$

Growth facts

Intuition:

- Growth is generated by aggregate capital accumulation
- Aggregate capital accumulation is determined by aggregate investment in the intermediate good sector
- We saw that the level of investment maximizes profits of the intermediate producers. As intermediate producers have market power, they maximize profits by taking into account the effects of their production on the demand for their products
- Their market power is smaller if there is a higher degree of substitution between varieties, i.e., ε high
- This implies that aggregate capital accumulation depends on their market power

$$\dot{K} = I - (\mu A + \delta) K$$

References

See https://pmbbrito.github.io/cursos/master/ec/2122/endogenous_NK_2122.pdf