R&D and growth: the variety expansion model

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Core assumptions

of the version of the model presented next

- **Technical progress** is materialized in the expansion of intermediate goods, i.e., creation new industries (”horizontal innovation”)
  - technical progress takes the form of an expansion in the number (variety) of products
  - new varieties are new intermediary goods (not new consumer goods as in the ”love-for-variety” models)

- R&D activity by an entrant: production of ideas that generate a new good (and a new industry)
- R&D technology: lab-equipment (not knowledge-driven)
- R&D value: If successful the **entrant becomes a monopolist** in its market (forever)
- **Free-entry condition**: R&D is only done if the value of R&D covers its costs

Simplifying assumptions

of the version of the model presented next

- There is no capital accumulation
- Population is constant and exogenous
- The only driver for growth is the increase in TFP which takes the form of an expansion in the number of products

Environments

We consider two environments:

- decentralized economy: R&D expenditures and profits are an externality
- centralized economy: R&D costs and benefits are internalized by the fiscal policy

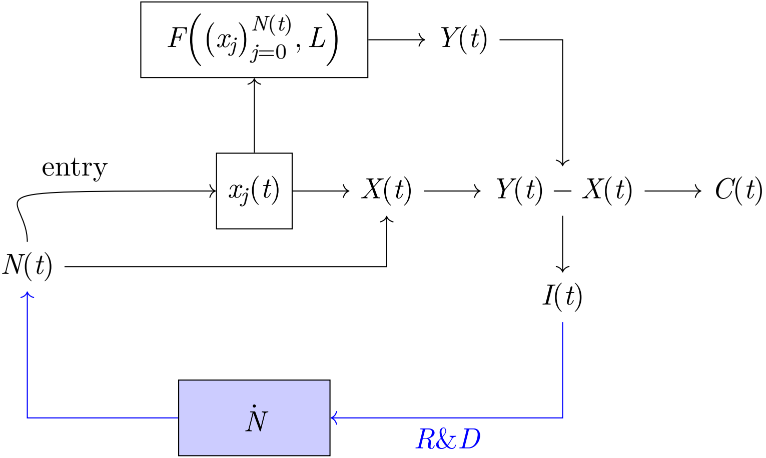
Results: implication for growth

- Without capital accumulation, **growth is generated by the expansion in varieties**
- The rate of growth depends on the **barriers to entry into R& D**
- The **decentralized economy is not Pareto optimal**: there is an externality not internalized
- The rate of return generated by R&D activities is lower in a decentralized than in a related centralized economy, and therefore the rate of growth will be higher if the externality is internalized

Economic structure

- Environment (new Keynesian):
  - there are two sectors: a competitive final good sector and a continuum of monopolistically competitive intermediate goods sectors
  - there is entry by creation of a new intermediate good product (=industry)
- Technology:
  - final good production uses labor and a continuum of intermediate goods
  - intermediate goods are the only reproducible inputs
  - the dynamics of output is generated by the variations in the number of of intermediate inputs (varieties) which is the result of successful R&D (research and development=)

The mechanics of the model



Decentralized (market) economy

Decentralized economy

The structure of the model

- 1. Household's problem (CP)
- 2. Final producer problem (FGPP)
- 3. Producers of intermediate goods (incumbents and entrants) (IGPP)
- 4. Aggregation, balance sheet and market clearing conditions
- 5. DGE model (DGE)

1. The household's problem

- Earns labor and capital income, consumes a final product and save
- own firms (final good and intermediate good producers)
- The problem

$$\max_{(C(t))_{t\in[0,\infty)}} \int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \ \theta > 0$$

$$\begin{aligned} &\text{subject to} \\ &\dot{W} = \omega(t)L + r(t)W(t) - C(t) \\ &W(0) = W_0, \text{initial condition} \\ &\lim_{t\to\infty} e^{-R(t)} W(t) \geq 0 \text{ NPG condition} \end{aligned} \tag{CP}$$

where $R(t) = e^{\int_0^t r(s)\,ds}$ is the market discount factor

1. The household's problem

- The first order conditions are

$$\dot{C} = \frac{C}{\theta}\left(r(t) - \rho\right) \quad (1)$$

$$\dot{W} = \omega(t) L + r(t) W(t) - C(t) \quad (2)$$

1. The household's problem

Proof:

- The Hamiltonian function is

$$H(C, W, Q) = \frac{C^{1-\theta} - 1}{1-\theta} + Q\,(\omega(t)\,L + r\,W - C)$$

- the f.o.c are

$$\frac{\partial H}{\partial C} = 0 \iff C(t)^{-\theta} = Q(t)$$

$$\dot{Q} = \rho\,Q - \frac{\partial H}{\partial W} \iff \dot{Q}(t) = (\rho - r(t))\,Q(t)$$

- then

$$-\theta\,\frac{\dot{C}}{C} = \frac{\dot{Q}}{Q} = \rho - r(t) \Rightarrow \text{equation}\,(1)$$

- Observe that $r(t)$ is endogenous and is determined at the general equilibrium

2. Producer of the final good

- Buys labor services and intermediate goods and sells a final good
- Production function: Dixit and Stiglitz (1977)

$$Y(t) = AL^{1-\alpha} \int_0^{N(t)} x(j,t)^{\alpha} dj, \; 0 < \alpha < 1$$

  α share of intermediate products j are symmetric
- L labor input
- $(x(j,.))_{j \in [0, N(t)]}$ intermediate inputs, non-storable,
- $N(t)$ number of varieties
- Should display CRS (constant returns to scale)

2. Producer of the final good

- Producer profit:

$$\pi^p(t) = Y(t) - \omega(t)L - \int_0^{N(t)} P(j,t)\, x(j,t)\, dj$$

$P(j,t)$ relative price of the intermediate good (final good price = 1)

- The problem:

$$\max_{L,(x(j,t))_{j\in[0,N(t)]}} \pi^p(t) \qquad \text{(FGPP)}$$

- Obs: it is a price taker in all markets (decision variables: quantities)

2. Producer of the final good

First order conditions:

- demand for labor

$$L^d = (1-\alpha)\frac{Y(t)}{\omega(t)} \tag{3}$$

- demand for intermediate goods

$$x^d(j,t) = \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,\ j \in [0, N(t)] \tag{4}$$

2. Producer of the final good

Proof:

- The profit is

$$\pi^p(L,[x]) = AL^{1-\alpha}\int_0^N x(j)^\alpha \, dj, -\omega\, L - \int_0^N P(j)\, x(j)\, dj$$

- F.o.c for labor

$$\frac{\partial \pi^p(L,[x])}{\partial L} = 0 \iff (1-\alpha)\frac{Y}{L} = \omega \Rightarrow \text{equation}\,(3)$$

- F.o.c for input j

$$\frac{\delta \pi^p(L,[x])}{\delta x(j,.)} = \alpha\, A\, L^{1-\alpha}\, x(j) - P(j) = 0 \Rightarrow \text{equation}\,(4)$$

3. Producers of intermediate goods

- Does R&D before entry and, if successful, after entry, produces a new variety (to be bought by final producers)
- **Decision process** for the introduction of a new variety
  - before entry: perform R& D
  - entry decision: free entry condition
  - after entry: decide on the price of variety j, $P(j,t)$ (upon entry)
- **Solution** to the problem: work **backwards**
  - (1): we determine the pricing policy assuming if there is entry (incumbent's problem)
  - (2): we determine entry (by using the free entry condition)

3. (1) Producer of intermediate good $j \in (0, N(t)]$

Price decision after entry

- The profit of the producer of a variety $j \in (0, N(t)]$ is

$$\pi(j,t) = (P(j,t) - MC)x(j,t)$$

assuming a symmetric cost of production equal to MC

- where $x(j,t) = x^d(j,t)$ (solution of the FGPP)
- Then the **profit after entry** is

$$\pi(j,t) = (P(j,t) - MC)\left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L,$$

- As it is a monopolist in the market for good j, its problem is

$$\max_{P(j,t)} \pi(j,t)$$

3. (1) Producer of intermediate good $j \in (0, N(t)]$

Price decision after entry

- The first order condition:

$$P^*(j,t) = \frac{MC}{\alpha} = \mu\, MC\, \forall (j,t) \tag{5}$$

where $\mu = 1/\alpha > 1$ is the mark up (of price over the marginal cost)

- Proof:

$$\pi(j) = \big(P(j) - MC\big) \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L$$

- then

$$\frac{\partial \pi(j)}{\partial P(j)} = \left(\frac{\alpha A}{P(j,t)}\right)^{\frac{1}{1-\alpha}} L \Big(1 - \frac{(P(j) - MC)}{(1-\alpha)} \frac{1}{P(j)}\Big) = 0$$

3. (1) Producer of intermediate good $j \in (0, N(t)]$

Demand and profit product j

- From now on we set $MC = 1$
- Then the demand for any variety is **symmetric** (i.e, equal to all industries)

$$x^*(j,t) = x^* = \left(\alpha^2 A\right)^{\frac{1}{1-\alpha}} L, \text{ for any } j \in [0, N(t)]$$

- the profit is also **symmetric** across industries and constant

$$\pi^*(j,t) = \pi^* = \left(\frac{1-\alpha}{\alpha}\right) L \left(A\alpha^2\right)^{\frac{1}{1-\alpha}} > 0$$

- Obs: $\mu - 1 = \dfrac{1-\alpha}{\alpha}$ (monopoly rent)

3. (1) Producer of intermediate good $j \in (0, N(t)]$

Profits after entry

- Defining the output per variety by

$$y^v \equiv (A\alpha^{2\alpha})^{\frac{1}{1-\alpha}} L \tag{6}$$

- Then

$$\pi^* = \alpha(1-\alpha)y^v$$

  (pure-) profits are positive, symmetric and constant in time.

3. (2) Decision to entry

Value of doing successful R&D and entry

- The **value from producing a successful variety** j, if it is introduced (by entry) at time t, is a monopoly rent forever

$$v(j,t) = \max_{(P(j,s))_{s\in[t,\infty)}} \int_t^\infty \pi(j,s)\, e^{-R(s)}\, ds \qquad \text{(IGPP)}$$

- where the **market** discount factor is time-variying

$$R(s) = \int_t^s r(\tau)\, d\tau$$

- Introducing the profit for an incumbent in industry j, $\pi(j,t) = \pi^*$, at the optimum we have

$$v^*(j,t) = v^*(t) = \pi^* \int_t^\infty e^{-R(s)}\, ds$$

- taking a time derivative yields (using the Leibniz integral rule)

$$\dot{v}(t) = -\pi^* + r(t)\, v(t) \qquad (7)$$

3. (2) Decision to entry

Cost of doing R&D

- **Lab-equipment assumption**: R&D is an activity using the final product as an input
- Costs of entry: assuming a linear and symmetric R&D technology

$$I(j,t) = \eta y^v(t)$$

  the cost of entry is proportional to output per variety j

3. (2) Decision to entry

Free entry condition

- **Free entry condition** in the market for variety j there is entry up to the point in which benefits are equal to the costs of entry
- Therefore, the equilibrium entry condition is

$$v(j,t) = I(j,t)$$

- Then, taking $v(j,t) = v^*(t)$ and $I(j,t) = \eta\, y^v$

$$\boxed{v^* = \eta y^v}$$

- Observation: from (6) v is a constant which implies $\dot{v} = 0$

3. (2) Decision to entry

Arbitrage condition

- Because v^* is a constant, then from (7) (and $\dot{v} = 0$)

$$\pi^* = rv^*$$

- Then: **arbitrage between entry and investing in existing firms**

$$\boxed{r(t) = r^* = \frac{\pi^*}{v^*} = \frac{\alpha(1-\alpha)}{\eta} = \frac{\alpha^2(\mu - 1)}{\eta}} \tag{8}$$

therefore, the interest rate is a constant and increases with the monopoly rent and decreases with the cost of doing R&D (barriers to entry)

4. Aggregation and consistency conditions

Aggregate output

- Using $x^*(j,t) = (\alpha^2 A)^{\frac{1}{1-\alpha}} L$ then

$$\boxed{Y(t) = AL^{1-\alpha} \int_0^{N(t)} (x^*)^\alpha dj = y^v N(t)}$$

  because

$$A \left(A\alpha^2\right)^{\frac{\alpha}{1-\alpha}} L = \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} L = y^v$$

- Observation: output is a linear function of the number of varieties
- We also obtain

$$X(t) = \int_0^{N(t)} x^* dj = x^* N(yt) = \alpha^2 y^v N(t)$$

- Therefore net output (value added) is

$$Y(t) - X(t) = \left(1 - \alpha^2\right) y^v N(t)$$

4. Aggregation and consistency conditions

Consistency conditions

- The rents generated by R&D distributed to consumers who own firms

$$W(t) = \int_0^{N(t)} v(j,t)\,dj = v^* N(t) = \eta y^v N(t)$$

- Substituting in the budget constraint (equation (2))

$$\dot{W} = \omega L + rW - C \Leftrightarrow \eta y^v \dot{N} = (1-\alpha)(1+\alpha) y^v N - C$$

- because $\dot{W} = \eta y^v \dot{N}$
- from equation (3): $\omega L = (1-\alpha) Y = (1-\alpha) y^v N$
- from equation (8) $rW = \dfrac{\alpha(1-\alpha)}{\eta} \eta y^v N$

General equilibrium

Market equilibrium

- Equilibrium condition

$$Y(t) = C(t) + I(t) + X(t)$$

- We derived $Y(t) - X(t) = (1 - \alpha^2) y^v N(t)$
- Aggregate investment in R&D

$$I(t) = \int_0^{\dot{N}(t)} I(j,t)\, dj = \int_0^{\dot{N}(t)} \eta y^v dj = \eta y^v \dot{N}(t)$$

- Therefore, we get same relationship

$$(1 - \alpha^2) y^v N(t) = C(t) + \eta y^v \dot{N}(t)$$

General equilibrium: alternative representation

- If we define the capital in this economy as $K(t) = W(t)$. Then $K(t) = \eta y^v N(t)$, $\omega L = \frac{(1-\alpha)}{\eta} K$ and $rW = \frac{\alpha(1-\alpha)}{\eta} K$
- the budget constraint becomes

$$\dot{K} = \frac{(1-\alpha)(1+\alpha)}{\eta} K - C = A^v K - C$$

- which implies that the model has a AK structure, where $A^v = A^v(\alpha, \eta) = \dfrac{(1-\alpha)(1+\alpha)}{\eta}$, where clearly

$$\frac{\partial A^v}{\partial \alpha} < 0, \ \frac{\partial A^v}{\partial \eta} < 0$$

which means that A^v is a positive function of the markup, $\mu = 1/\alpha$: an increase in the markup and a reduction in the barriers to entry increase the productivity of capital

The equilibrium in the decentralized economy

- the DGE in levels

$$\boxed{\begin{aligned}\dot{K} &= A^v K - C \\ \dot{C} &= \frac{C}{\theta}(r_d - \rho),\end{aligned}} \tag{DGE}$$

- where the real rate of return and the TFP are

$$r_d = r(\alpha, \eta) = \frac{\alpha(1-\alpha)}{\eta}$$

$$A^v = A^v(\alpha, \eta) = \frac{(1-\alpha)(1+\alpha)}{\eta}$$

Separating trend from transition

- We decompose the variables

$$C(t) = c(t)e^{\gamma t},\ K(t) = k(t)e^{\gamma t}$$

- the DGE in detrended variables

$$\boxed{\begin{aligned} \dot{k} &= \left(A^v - \gamma\right) k - c \\ \dot{c} &= \frac{c}{\theta}(r - \rho - \theta\gamma) \end{aligned}} \qquad \text{(DGE detrended)}$$

The long run growth rate

Decentralized economy

- the long run growth rate

$$\boxed{\gamma_d = \frac{r_c - \rho}{\theta} = \frac{1}{\theta}\left(\frac{\alpha(1-\alpha)}{\eta} - \rho\right)}$$

  is a negative function of the cost of entry η (i.e, **barriers to R&D reduce growth**)

- the long run level for per capita GDP is

$$\bar{y} = y^v(A, L)\frac{n(0)}{L} = \left(A\alpha^{2\alpha}\right)^{\frac{1}{1-\alpha}} n(0)$$

- there is no transitional dynamics

Centralized (Pareto) economy

Centralized economy

- Consider a social planner solving the problem

$$\max_{(C(t))_{t\in[0,\infty)}} \int_0^{\infty} \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \; \theta > 0$$
$$\text{subject to}$$
$$\dot{K} = A^v K - C \tag{OP}$$

- applying the Pontriyagin principle and decomposing the variables we get

$$\begin{aligned} \dot{k} &= (A^v - \gamma)\, k - c \\ \dot{c} &= \frac{c}{\theta}(r_c - \rho - \theta\gamma) \end{aligned} \tag{OP detrended}$$

where

$$\boxed{r_c \equiv \frac{1-\alpha^2}{\eta}}$$

The long run growth rate

Centralized economy

- the long run growth rate

$$\boxed{\gamma_c = \frac{1}{\theta}\left(\frac{1-\alpha^2}{\eta} - \rho\right) > \gamma_d = \frac{1}{\theta}\left(\frac{(1-\alpha)\alpha}{\eta} - \rho\right)}$$

- the long run growth rate in the centralized economy is higher than in the decentralized economy
- this means that the decentralized economy is not Pareto optimal: there is an externality generated by the R&D activity that is not internalized in a decentralized economy

Implementing an optimal policy in a decentralized economy

Policy implications

- In the decentralized setting the government introduces a tax/subsidy on the return on capital applied/financed by a lump-sum expenditure/tax
- in the first case (tax/expenditure) τ and G are positive and in the second (subsidy/tax) they are negative

the budget constraint for the household becomes

$$\dot{W} = \omega L + (1 - \tau) r W + G - C$$

- Assume a budget balanced rule

$$\tau r(t) W(t) = G(t)$$

Policy implications

- This implies that the rate of growth becomes

$$\gamma_d = \frac{1}{\theta}\left(\frac{(1-\tau)\alpha(1-\alpha)}{\eta} - \rho\right)$$

- to internalize fully the externality we should have $(1-\tau)r_d = r_c$ which implies $\gamma_d = \gamma_c$, that is

$$(1-\tau)\frac{\alpha(1-\alpha)}{\eta} = \frac{(1+\alpha)(1-\alpha)}{\eta}$$

- then the **optimal policy** that would implement a Pareto DGE would be: a subsidy whose rate should be equal to the markup $-\tau = \mu = \dfrac{1}{\alpha}$

References

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