Automation and growth

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Automation growth and the labor market

- There is a reduction in the labor share of income in several countries Ex. US also in Europe (see page 10)
- There is also an increase in the number of robots particularly in Asia in all industries
- Are the two processes related ?

Robots and machines

- Robots have a peculiar nature:
  - they are generated by a production process similar to machines
  - they are financed in a similar way as machines
  - but they are more substitutable to labor than to machines
- Next we show that the inclusion of robots can generate endogenous growth in a Ramsey-like model
- And, potentially, a permanent reduction in the labor share in GDP

The macroeconomic constraint

- Assumption:
  - machines are Edgeworth complements to labor and robots
  - labor and robots are Edgeworth substitutes
- The production function

$$Y = M^{\alpha}\left(A_n N + L\right)^{1-\alpha},\ 0 < \alpha < 1$$

where $Y =$ output, $M =$ input of machines, $N =$ input of robots, and $L =$ labour input, A_n is the specific productivity of robots

The macroeconomic constraint

- Assumption: savings are proportionally allocated to investment in machines, θ, and to robots $1-\theta$
  - Distribution of savings to machines

$$\theta\, S = \dot{M} + \delta M$$

    δ = depreciation rate
  - Distribution of savings to robots

$$(1-\theta)\, S = \dot{N} + \delta N$$

- Then

$$S = \dot{N} + \delta N + \theta S = \dot{M} + \dot{N} + \delta(M+N) = \dot{K} + \delta K$$

where the stock of capital is

$$K \equiv M + N$$

The macroeconomic constraint

- Equilibrium in the good's market

$$Y = C + S \iff Y = C + \dot{K} + \delta\, K$$

- Assumption: zero population growth

$$\dot{L} = 0$$

The macroeconomic constraint

- Capital intensity

$$k(t) \equiv \frac{K(t)}{L(t)}$$

- Fraction of machines in the total capital stock

$$\mu \equiv \frac{M}{K} \in [0,1]$$

- The equilibrium condition is equivalent to

$$\dot{k} = \left(\mu\, k\right)^{\alpha} \Big(A_n\,(1-\mu)\,k + 1\Big)^{1-\alpha} - c - \delta\, k$$

where $c \equiv C/L$ (per-capita consumption)

A Ramsey problem: efficient capital accumulation

Find the consumption and capital allocation between machines and robots to solve

$$\max_{c(\cdot),\mu(\cdot)} \int_0^\infty \ln\left(c(t)\right) e^{-\rho t}\, dt,\ \ \rho > 0$$

$$\begin{aligned}
&\text{subject to} \\
&\dot{k} = y(\mu, k) - c - \delta\, k \\
&0 \le \mu(t) \le 1 \\
&k(0) = k_0 \text{ given} \\
&\lim_{t\to\infty} e^{-\rho t}\, k(t) \ge 0
\end{aligned} \tag{P1}$$

where per-capita output is

$$y(\mu, k) = \left(\mu\, k\right)^\alpha \Big(A_n\,(1-\mu)\,k + 1\Big)^{1-\alpha}$$

The optimal share of machines in capital

- the optimal share of machines depends on the level of the capital stock

$$\mu^* = \begin{cases} 1 & \text{if } 0 < k \leq k_m \\ \dfrac{\alpha\,(1 + A_n\,k)}{A_n\,k} & \text{if } k > k_m \end{cases}$$

where

$$k_m \equiv \frac{1}{A_n}\left(\frac{\alpha}{1-\alpha}\right).$$

- there are two regimes: no automation (for low level of capital) and automation (for higher level of capital)

The optimal share of machines in capital

Proof:

- The Hamiltonian is

$$H(c, \mu, k, q) = \ln c + q \left(y(k, \mu) - c - \delta\, k \right)$$

  subject to the constraint $0 \leq \mu \leq 1$

- The optimality condition for μ, assuming that it has no constraint, is

$$q \frac{\partial y}{\partial \mu} = 0$$

  where

$$\frac{\partial y}{\partial \mu} = \frac{y}{\mu} \left(\frac{A_n\,(\alpha - \mu)\,k + \alpha}{A_n\,(1 - \mu)\,k + 1} \right)$$

- Then, because $q > 0$ it holds if and only if

$$\mu = \alpha \left(1 + \frac{1}{A_n\,k} \right)$$

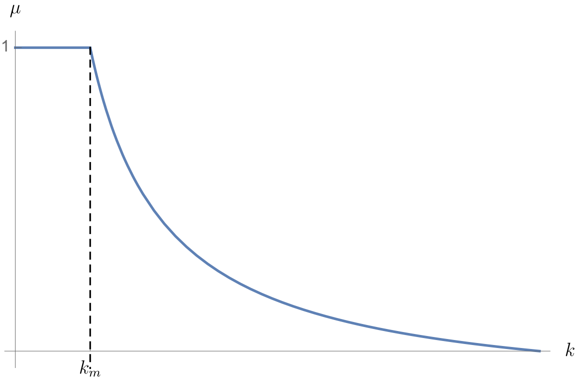

Figure: The optimal capital allocation function $\mu^*(k)$ where $M = \mu^* K$

The optimal share of machines in capital

Proof:

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where

$$\frac{\partial y}{\partial \mu} = \frac{y}{\mu}\left(\frac{A_n\,(\alpha - \mu)\,k + \alpha}{A_n\,(1-\mu)\,k + 1}\right)$$

- Then, because $q > 0$ it holds if and only if

$$\mu = \alpha\left(1 + \frac{1}{A_n\,k}\right) > 0$$

as we see $\mu'(k) < 0$ the proportion of machines (robots) in capital decreases (increases) with k

The optimal share of machines in capital

Proof (cont.):

- $\mu = 1$ if and only if $k = \alpha / \big(A_n . (1 - \alpha) \big) = k_m$
- therefore
  - $\mu \leq 1$ (non-binding constraint) if and only if $k \geq k_m$
  - $\mu = 1$ (binding if $k < k_m$)

Two regimes

- **There are two regimes**
  - For low levels of capital there are no robots
  - For higher levels of capital there robots
- Optimal output in the two regimes (no robots, with robots)

$$Y(k) = \begin{cases} k^{\alpha} & \text{if } 0 < k \leq k_m \\ \phi(A_n)\left(A_n\, k + 1\right) & \text{if } k > k_m \end{cases} \tag{1}$$

  where $\phi(A_n) \equiv A_n^{-\alpha}\, \alpha^{\alpha}\, (1-\alpha)^{1-\alpha}$
- Proof: for $\mu = 1$ it is easy, but for $0 < \mu < 1$ we have

$$\begin{aligned} y &= (\mu\, k)^{\alpha} \left(A_n\, (1-\mu)\, k + 1\right)^{1-\alpha} \\ &= \left(\alpha\left(\frac{1 + A_n\, k}{A_n}\right)\right)^{\alpha} \left(A_n(1-\alpha)k - \alpha + 1\right)^{1-\alpha} \\ &= \alpha^{\alpha}\, (1-\alpha)^{1-\alpha} \left(1 + A_n\, k\right) \end{aligned}$$

Two regimes

- Rate of return for the two regimes

$$R(k) = \begin{cases} \alpha\, k^{\alpha-1} & \text{if } 0 < k \le k_m \\ \tilde{r} & \text{if } k > k_m \end{cases} \tag{2}$$

where

$$\tilde{r} = A_n\, \phi(A_n) = A_n^{1-\alpha}\, \alpha^{\alpha}\, (1-\alpha)^{1-\alpha}.$$

- Proof: in $R(k) = \dfrac{\partial y(k,\mu)}{\partial k}$, where

$$\frac{\partial y(k,\mu)}{\partial k} = \frac{y}{k} \left(\frac{\alpha + A_n\,(1-\mu)\,k}{1 + A_n\,(1-\mu)\,k} \right)$$

we make the same substitutions as for the production functions

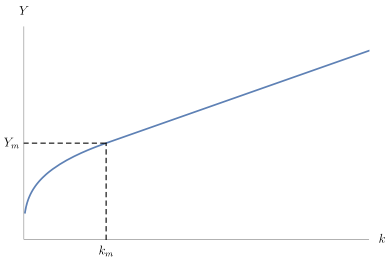

Figure: The optimal output and real return functions $Y(k)$ and $R(k)$.

The optimum dynamics

- The optimum path $\Big(k(t), c(t)\Big)_{t\in[0,\infty)}$ is the solution of

$$\begin{cases} \dot{k} &= Y(k) - c - \delta\, k \\ \dot{c} &= c\Big(R(k) - (\rho+\delta)\Big) \\ & k(0) = k_0 \\ & \lim_{t\to\infty} \dfrac{k(t)}{c(t)} e^{-\rho t} = 0 \end{cases} \tag{MHDS}$$

- where $k(t)$ and $c(t)$ are level variables.

Steady state

There is a critical value for productivity

$$A_n^* \equiv \left(\frac{\rho + \delta}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} \tag{3}$$

1. if $A_n < A_n^*$ then there is one steady state (k^*, c^*) such that $k^* < k_m$;
2. if $A_n = A_n^*$ then there is a steady state (k_m, c^*);
3. if $A_n > A_n^*$ then there are no steady states

No automation steady state

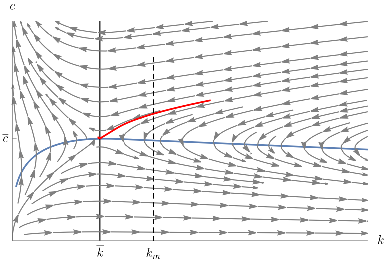

Figure: Phase diagram for $A_n < A_n^*$.

- if $A_n < A_n^*$ (high costs in using robots)
- there is no long run growth (Ramsey case)

Automation

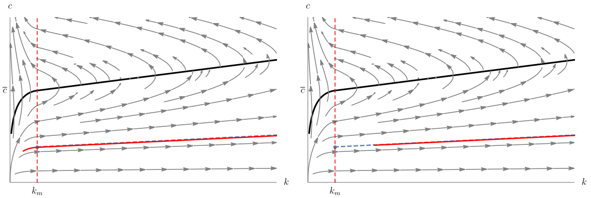

Figure: Phase diagram for $A_n > A_n^*$ starting in region 1 and 2.

- if $A_n > A_n^*$ (high costs in using robots)
- there is long run growth (as in the AK model)
- independently of the level of k_0
- the long-run growth rate is for $u(c) = \ln(c)$

$$\gamma = \tilde{r}(A_n) - \rho - \delta$$

Modified AK case

- Assume that $A_n > A_n^*$ and $k_0 > k_m$
- Solving the problem (MHDS) we obtain the solution for the capital stock

$$k(t) = k_0 \, e^{\gamma \, t} + \frac{\phi(A_n)}{\rho + \gamma}\left(e^{\gamma \, t} - 1\right)$$

- Exercise: prove this
- Therefore, if $\gamma > 0$ then

$$\lim_{t \to \infty} k(t) = \infty$$

The share of labor in national income

- The share of labor in the GDP is

$$\omega = \frac{W\,L}{Y} = \frac{\partial\,Y}{\partial L}\frac{L}{Y} = \frac{1-\alpha}{A_n\,(1-\mu)\,k+1}$$

  where W is the wage rate

- In the optimum, if $0 < \mu(k) < 1$ it becomes

$$\omega(t) = \frac{1}{1 + A_n\,k(t)}$$

- Therefore, as $\lim_{t\to\infty} k(t) = \infty$ then the share of labor in national income converges to zero **asymptotically**

$$\lim_{t\to\infty} \omega(t) = 0$$

  (under the previous conditions: $A_n > A_n^*$ and $k_0 > k_m$)

Final remarks

- Although this model refers to robots, it is more general.
- It shows a potential emergence of endogenous growth from a Ramsey type model.
- Possibly, industrial revolutions start by introducing tools that are substitutable with labor, at least in a first phase.
- Although it is a simple model it raises the question: does the reduction of the labor share is a permanent consequence of automation ?

Final remarks

- Off course, there will be other mechanisms that would avoid this type of development:
- For instance if we introduce high-skill labor which would be complementary with machines and robots, will this result old ?
- Is this result dependent upon the particular production function we have introduced ? Even if we assume that robots and labor are substitutable ?