Taxes, public debt and economic growth

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Brief historical evidence

- Historically and across countries there is a positive correlation between "state capacity" and development
- The construction of the fiscal state has been historically related to war financing (an episode for Portugal 1640)
- However, war financing also had the effect of building up the "welfare state" and creating most of the institutions we know today (see The great leveller)
- This implied that taxes and public debt increased to unprecedented levels
- Maybe there is no growth model that makes sense without considering some type of public intervention

The state and the economy

- We can see the **state** as a sector which has the "technology" of dealing with **aggregate allocations**.
- This covers a big number of issues: setting up an institutional framework that allocates asset and income rights, producing and/or supplying public goods (infrastructure, education, health systems, social security systems), pooling information and economic decision, changing adjustments costs of several types, etc;
- However, when its intervention generates pecuniary costs it should finance those costs, which means again that, given its size, it will imply effects in **aggregate incentives**. (v.g., crowding out - for instance it is shown here that public finances are a key explanation of the change in the capital structure of firms),

This lecture

- There are zillions of different models we could think about for addressing the role of government on growth (we already saw quite a few):
- In this lecture we address the consequences of government financing, through debt and/or taxes, on the rate of growth
- We assume that government expenditures generate an externality in production, which could be seen as investment in infrastructures (of a flow nature). However, the level of expenditures and the type of financing have an effect on the growth rate.
- The question is: **under which circumstances the need to finance expenditures will not jeopardize the positive effect of the externality, in the long run ?**

Assumptions

- Production of goods involves an **externality** associated to the services provided by the state;
- Services provided by the public sector are a **function of the government expenditures**;
- The government has to finance expenditures by **taxes and/or debt**
- Financing is constrained by the **government budget constraint** (GBC)
- We assume that the government uses a **rule** of keeping the debt over the GDP ratio (B/Y) constant

Workings of the model

- There are two extreme **financing strategies**: tax finance (debt ratio equal to zero), debt finance (tax rate equal to zero)
- Taxes will distort the incentives to capital accumulation: this implies the model has externalities that are not internalized; if the GBC does not need to be permanently balanced, this changes the rate of growth of the economy
- However, if there is a cap on government borrowing there is a **feedback mechanism** requiring the increase in taxes

Conclusions

- Government expenditures have a **positive** effect on the growth rate
- However, the **type of financing matters** for the long run level of the ratio G/Y:
  - with **tax financing**, taxes have an ambiguous effect on the growth rate (it increases the long run interest before taxes but the effect on the net rate is ambiguous)
  - with **debt financing**, such that B/Y is constant, we can have a negative effect on growth because the implied sustainability requires that the long-run interest rate should be reduced
- The final outcome is: there is a (kind of) Laffer curve

The model

Private sector

- Chooses $(C(t))_{t\geq 0}$ to maximize

$$\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta}e^{-\rho t}dt$$

  C is consumption

- subject to the budget constraint

$$\dot{K}+\dot{B}=(1-\tau)\left(r(t)B(t)+Y(t)\right)-C(t)$$

  B government bonds, K private capital, τ tax rate

- non-Ponzi game condition

$$\lim_{t\to\infty}(K(t)+B(t))e^{\int_0^t r(s)ds}\geq 0.$$

- I am assuming: there is only an **income tax with a flat rate** (if K includes human capital a labor tax is implicitly assumed along with capital tax)

The model

Private sector: first order conditions

- Euler equation

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(t) - \rho}{\theta} \tag{1}$$

- Budget constraint

$$\dot{K} + \dot{B} = (1-\tau)\left(r(t)B(t) + Y(t)\right) - C(t) \tag{2}$$

- Transversality condition

$$\lim_{t\to\infty} C(t)^{-\theta}\left(K(t) + B(t)\right)e^{-\rho t} = 0.$$

The model

Production and arbitrage in financial markets

- Production technology

$$Y(t) = K(t)^{\alpha} G(t)^{1-\alpha},\ 0 < \alpha < 1 \tag{3}$$

  G government expendititures

- arbitrage condition between capital and government debt

$$r(t) = \alpha K(t)^{\alpha-1} G(t)^{1-\alpha}. \tag{4}$$

The model

The government

- GBC: government budget constraint

$$\dot{B} = \underbrace{r(t)B(t)}_{\text{interest payments}} + \underbrace{G(t) - \tau\left(r(t)B(t) + Y(t)\right)}_{\text{primary deficit}} \tag{5}$$

- Policy rule: the government keeps the ratio of debt over GDP constant

$$B(t) = \bar{b}\,Y(t) \tag{6}$$

where $\bar{b}$ is a number (Maastricht rule 60%)

The aggregate constraint

- Consolidating the private and government constraints yields the aggregate constraint of the economy

  $$\dot{K} = Y(t) - C(t) - G(t)$$

  which turns out to be equal to the equilibrium in the goods' market

  $$Y(t) = C(t) + I(t) + G(t)$$

  where $I = \dot{K}$

The DGE

Is represented by four equations:

- The Euler equation

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r(K,G) - \rho}{\theta} \tag{7}$$

- the economic-wide equilibrium condition

$$\dot{K} = Y(K,G) - C - G \tag{8}$$

- the GBC

$$\dot{B} = r(K,G)B + G - \tau\left(r(K,G)B + Y(K,G)\right) \tag{9}$$

- the policy rule

$$B = \bar{b}Y \tag{10}$$

Transforming the DGE

New variables

- In order to reduce the dimensionality of the model, define two variables
  - The ratio of government expenditures over the GDP

$$g \equiv \frac{G}{Y}$$

  - The ratio of consumption over the capital stock

$$z \equiv \frac{C}{K}$$

The transformed DGE

GDP and interest rate

- Defining theshare of public expenditure in the GDP

$$g \equiv \frac{G}{Y}$$

- The GDP is

$$\boxed{Y = A(g)K, \text{ for } A(g) \equiv g^{\frac{1-\alpha}{\alpha}}, \; A^{'} > 0}$$

Externality effect: an increase in g increases the TFP (A)

- The real interest function is a positive function of g

$$\boxed{r(g) = \alpha A(g),}$$

$r^{'} > 0$ because $A^{'} > 0$ Externality effect: an increase in g increases the rate of return of capital

The transformed DGE

The rate of growth

- Then the rate of growth of GDP is (taking log-derivatives)

$$\gamma_y = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g}$$

Externality effect: an increase in g increases the rate of growth

The transformed DGE

The transformed aggregate constraint

- The Euler equation (instead of (7)) becomes

$$\frac{\dot{C}}{C} = \gamma(g) \equiv \frac{(1-\tau)r(g) - \rho}{\theta}$$

  where $\gamma(g)$ is the rate of growth of consumption

- The rate of growth for private capital (instead of (8)) is

$$\frac{\dot{K}}{K} = \gamma_k(g, z) \equiv (1-g)A(g) - z$$

  where $\gamma_k(g)$ is the rate of growth of the capital stock, and $z \equiv C/K$

- Then the rate of growth of z becomes

$$\frac{\dot{z}}{z} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K}$$

The transformed DGE

The transformed aggregate constraint

- Substituting

$$\frac{\dot{z}}{z} = \gamma_z(g, z) = \gamma(g) - \gamma_k(g, z) = z - z(g)$$

where

$$z(g; \tau) \equiv \frac{(\theta(1-g) - \alpha(1-\tau))\, A(g) + \rho}{\theta}$$

can be rationalized as a **Laffer curve**: it includes two effects:

  - a **positive** net effect of g on GDP (bigger for small values of g): $(1-g)A(g)$
  - a **negative** net effect of taxes over r which decreases the rate of growth: $(1-\tau)r(g) = (1-\tau)\alpha A(g)$

The transformed DGE

The dynamics of b

- Defining the government debt over GDP ratio by $b = \dfrac{B}{Y}$ we get

$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{Y}}{Y} = \gamma_b - \gamma_y$$

- where (from equation (9))

$$\frac{\dot{B}}{B} = \gamma_b(g) = (1-\tau)r(g) + \frac{G - \tau Y}{B} = (1-\tau)r(g) + \frac{g-\tau}{b}$$

- Then, the dynamics of public debt over GDP is driven by

$$\frac{\dot{b}}{b} = (1-\tau)r(g) + \frac{g-\tau}{b} - \frac{\dot{K}}{K} - \frac{1-\alpha}{\alpha}\frac{\dot{g}}{g} \qquad (11)$$

The transformed DGE

The dynamics of g

- **Assumption**: introducing the policy rule $b = \bar{b}$ then $\dot{b} = 0$
- Then, we solve equation (11) for $\dot{g}/g$

$$\frac{\dot{g}}{g} = \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - \frac{\dot{K}}{K}\right) =$$

$$= \frac{\alpha}{1-\alpha}\left((1-\tau)r(g) + \frac{g-\tau}{\bar{b}} - ((1-g)A(g) - z)\right) =$$

$$= \frac{\alpha}{1-\alpha}\left(z - \zeta(g)\right)$$

where

$$\zeta(g) \equiv \left((1-g) - \alpha(1-\tau)\right)A(g) + \frac{\tau - g}{\bar{b}}.$$

The DGE in detrended variables

- The DGE in detrended variables is

$$\begin{cases} \dot{z} = (z - z(g;\tau))\ z \\ \dot{g} = \frac{\alpha}{1-\alpha}\left(z - \zeta(g;\tau,\bar{b})\right)\ g \end{cases} \quad (12)$$

where the long run $z = C/K$ ratio for a constant g is a Laffer curve

$$z(g;\tau) \equiv \frac{(\theta(1-g) - \alpha(1-\tau))\, A(g) + \rho}{\theta}$$

and associated with the GBR and policy rule is

$$\zeta(g;\tau,\bar{b}) \equiv ((1-g) - \alpha(1-\tau))\, A(g) + \frac{\tau - g}{\bar{b}}$$

- for $g \in (0,1)$ and $z > 0$

The long run level of g

- In system (12) we set $\dot{z} = \dot{g} = 0$.
- The long run level of g is a function of τ and $\bar{b}$:

$$g^{ss} = g^{ss}(\tau, \bar{b}) = \{g : \; \Phi(g : \tau, \bar{b}) \equiv z(g; \tau) - \zeta(g; \tau, \bar{b}) = 0\}$$

where

$$\Phi(g : \tau, \bar{b}) \equiv \frac{(1-\tau)(\theta-1)}{\theta} r(g) + \frac{g-\tau}{\bar{b}} + \frac{\rho}{\theta}$$

- The steady state satisfies

$$g^{ss} \leq \tau \leq \frac{\rho}{\theta}\bar{b}$$

only if $\bar{b} > 0$ (the government is a net debtor)

The long run level of g

- Next we prove that $g_{\tau}^{ss} > 0$ and $g_{\bar{b}}^{ss} < 0$
- That is:
  - an increase in the tax rate allows for an increase in the public expenditures over the GDP
  - but a higher the debt rule $\bar{B}$ implies a decrease in the public expenditures over the GDP

The function $\Phi(g)$

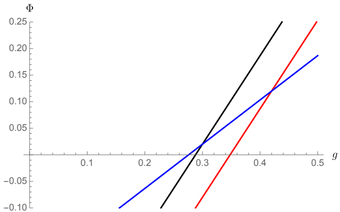

Figure: Function $\Phi(g, \tau, \overline{b})$ as a function of g. Increase in τ (red), increase in $\overline{b}$ (in blue)

The long run multipliers for g

Proof: the long run multipliers are

- for τ

$$\left.\frac{\partial g^{ss}}{\partial \tau}\right|_{g=g^{ss}} = -\left.\frac{\frac{\partial \Phi(g,.)}{\partial \tau}}{\frac{\partial \Phi(g,.)}{\partial g}}\right|_{g=g^{ss}}$$

- for $\bar{b}$

$$\left.\frac{\partial g^{ss}}{\partial \bar{b}}\right|_{g=g^{ss}} = -\left.\frac{\frac{\partial \Phi(g,.)}{\partial \bar{b}}}{\frac{\partial \Phi(g,.)}{\partial g}}\right|_{g=g^{ss}}$$

- Assuming that $(0 < \tau < 1 \text{ and } \theta > 1)$ then

$$\Psi \equiv \left.\frac{\partial \Phi(g,.)}{\partial g}\right|_{g=g^{ss}} = \frac{(1-\tau)(\theta-1)r^{'}(g^{ss})}{\theta} + \frac{1}{\bar{b}} > 0$$

The long run multipliers for g

The long run multipliers for g (evaluated at the steady state are):

- the effect of an increase in τ is positive

$$\left.\frac{\partial g^{ss}}{\partial \tau}\right|_{g=g^{ss}} = \frac{(1-\tau)r'(g^{ss}) - r(g^{ss})}{\theta\, \bar{b}^2\, \Psi} > 0$$

  if $(1-\tau)(1-\alpha) - \alpha g^{ss} > 0$; increases in the tax rate can finance increase in g

- the effect of increases of $\bar{b}$ on g^{ss} is negative

$$\left.\frac{\partial g^{ss}}{\partial \bar{b}}\right|_{g=g^{ss}} = \frac{(g^{ss} - \tau)\theta}{\theta\, \bar{b}^2\, \Psi} < 0$$

  because $\Phi(g, \tau) = 0$ only if $g^{ss} < \tau$; financing a higher level of debt competes with public expenditure

Long-run interest rate

- The long run interest rate is

$$\bar{r} = r(g^{ss}) = R(\tau, \bar{b})$$

- As $r^{'}(g) > 0$, then
  - an increase in the tax rate τ increases the long-run interest rate (through the externality of public expenditures channel): $\partial R/\partial \tau > 0$
  - an increase in the level of government debt $\bar{b}$ reduces the long run rate of return of capital (long-run crowding out effect): $\partial R/\partial \bar{b} < 0$

Long-run growth rate

- The long run growth rate is a positive function of the steady state g (g^{ss}):

$$\bar{\gamma} = \gamma(g^{ss}) = \frac{(1-\tau)r(g^{ss}) - \rho}{\theta}$$

- then the long-run growth rate depends on the public financing policy

$$\boxed{\bar{\gamma} = \gamma(\tau, \bar{b})}$$

- Next we prove that:
  - the growth effect of tax is ambiguous: γ_τ
  - the growth effect of debt is negative: $\gamma_{\bar{b}} < 0$

Long-run growth rate

Policy instruments and the growth rate

- The effect of the tax rate in the rate of economic growth

$$\frac{\partial \gamma}{\partial \tau} = \underbrace{-\frac{r(g^{ss})}{\theta}}_{-} + \underbrace{\frac{(1-\tau)}{\theta} r^{'}(g^{ss}) \frac{\partial g^{ss}}{\partial \tau}}_{+}$$

  is **ambiguous** because:
  - there is a **negative direct** effect (incentive effect on the reduction of savings)
  - and a **positive indirect effect** through financing the externality effect of g ion r (under the assumption if $(1-\tau)(1-\alpha) > \alpha g^{ss}$)
- depending on the level of τ one of the two effects can prevail: the first effect is stronger for very high or very low tax rates
- there is a tax rate that maximizes the rate of growth

Effect of the tax rate on the long run growth rate

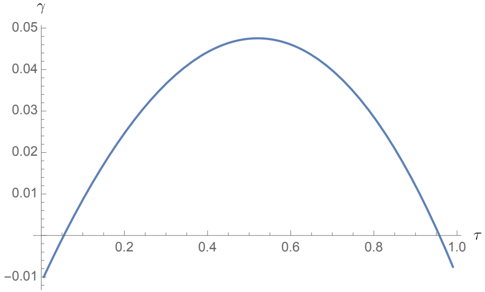

Figure: A Laffer curve (sort of): value of γ for different levels of the tax rate τ

Long-run growth rate

Policy instruments and the growth rate

- The effect of the debt ratio in the rate of economic growth

$$\frac{\partial \gamma}{\partial \bar{b}} = \frac{(1-\tau)}{\theta} r^{'}(g^{ss}) \frac{\partial g^{ss}}{\partial \bar{b}} < 0$$

  the effect on the growth rate is negative

- as a result of the crowding out effect: $\dfrac{\partial R}{\partial \bar{b}} < 0$

Transitional dynamics

- We can study the local dynamics of the model by finding the Jacobian of system (12) and evaluating it at the steady state (g^{ss}, z^{ss}) where $z^{ss} = z(g^{ss}) = \zeta(g^{ss})$
- the Jacobian is

$$J(g, z) = \begin{pmatrix} \frac{\alpha}{1-\alpha}\big(z - \zeta(g) - g\zeta'(g)\big) & \frac{\alpha}{1-\alpha} g \\ -z\,z'(g) & 2z - z(g) \end{pmatrix}$$

- evaluating at the steady state we have

$$J^{ss} = J(g^{ss}, z^{ss}) = \begin{pmatrix} -\frac{\alpha}{1-\alpha} g^{ss}\zeta'(g^{ss}) & \frac{\alpha}{1-\alpha} g^{ss} \\ -z^{ss}\,z'(g^{ss}) & z^{ss} \end{pmatrix}$$

- it has determinant

$$\det(J^{ss}) = \frac{\alpha}{1-\alpha}\, g^{ss}\, z^{ss}\Big(z'(g^{ss}) - \zeta'(g^{ss})\Big) = \frac{\alpha}{1-\alpha}\, g^{ss}\, z^{ss}\,\Psi > 0$$

Transitional dynamics

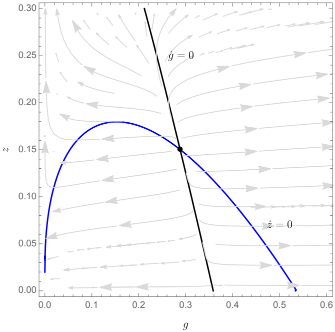

Figure: For $\theta = 1$, $\alpha = 0.7$, $\rho = 0.02$, $\tau = 0.3$ and $\bar{b} = 0.6$. The steady state is unstable: there is no transitional dynamics for $\Psi > 0$

Growth dynamics

- As the model has been solved for detrended variables
- the solution along the BGP is determined assuming that $g = g^{ss}$ and $z = z^{ss}$
- Therefore
$$Y(t) = A(g^{ss})e^{\bar{\gamma}t}$$

Growth facts

Effects of policy parameters τ and $\bar{b}$

- Long-run growth rate:
  - although a higher g increases the long run growth rate
  - the need to finance it generates a countervailing effect
    Conclusion: while debt financing reduces the long run growth rate, tax finance may reduce or increase the long run growth rate, although the positive effect dominates for intermediate levels of the tax rate.
- There is no transitional dynamics for $\Psi > 0$. The only dynamics is long-run dynamics. This is a consequence of setting B/Y constant.

A different policy rule

- Now assume that there is a rule on the government deficit (not on the government debt)

$$\frac{\dot{B}}{Y} \leq \beta \text{ EU level:3\%}$$

- To simplify assume that it is followed strictly $\frac{\dot{B}}{Y} = \beta$
- Show that now the dynamic system becomes

$$\begin{aligned} \dot{z} &= \left(z - z(g)\right) z \\ \dot{g} &= \left((1 - g)\, A(g) - z\right) (\beta + \tau - g) \end{aligned}$$

only the first equation changes

A different rule: conclusions

- Now the growth rate is

$$\bar{\gamma} = \frac{(1-\tau)\, r(g^{ss}) - \rho}{\theta}$$

  where $g^{ss} = \beta + \tau$
- The function $\gamma(\tau)$ has the same shape
- If β is higher the growth rate is higher as well
- There is transitional dynamics converging to the BGP
- But we have to check the transversality condition

Transitional dynamics

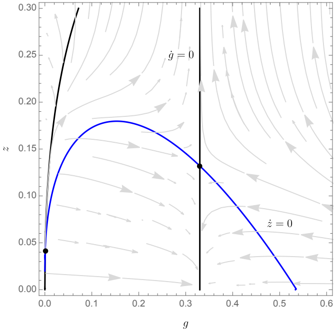

Figure: For $\theta = 1$, $\alpha = 0.7$, $\rho = 0.02$, $\tau = 0.3$ and $\beta = 0.03$. Now there are two steady states. There is transitional dynamics associated to the "high" growth steady state

References

- Barro (1990)

R. Barro. Government spending in a simple model of endogenous growth. *Journal of Political Economy*, 98: S103–S125, 1990.