

The Romer model: growth and externalities

Paulo Brito
pbrito@iseg.ulisboa.pt

30.3.2022

The Romer kK model

- ▶ It has the same assumptions as the AK model with the exception of the technology
- ▶ There are **externalities in production**

$$Y = AK^\alpha \mathbf{K}^\beta$$

where K is capital owned by firms and \mathbf{K} is a production externality

- ▶ **Assumption:** there are **decreasing** returns at the private level, $0 < \alpha < 1$, but there are **constant** returns at the aggregate level, $\alpha + \beta = 1$

The Romer kK model

- ▶ Meaning of the externality:
 - ▶ K can be seen as private capital and \mathbf{K} as public (or social) capital
 - ▶ Public or social capital increases the productivity of firms, even if firms cannot decide on its level
 - ▶ Only K is relevant for deriving firm's incentives to invest
- ▶ We will see that the government can induce the internalization of externalities through a tax/subsidy policy

The Romer kK model

- ▶ This introduces a distinction between
 - ▶ decentralized economy
 - ▶ centralized economy
- ▶ Next:
 - ▶ We solve the model for the **decentralized** economy (externalities not internalized)
 - ▶ We solve the model for a **centralized** economy (externalities internalized)
 - ▶ Discuss optimal policy: is it possible to design an **optimal policy in a decentralized economy**, that is a policy that **induces internalization** of externalities ?

Decentralized economy

Decentralized economy

Representative agent's problem

- ▶ The representative agent problem

$$\max_{\{C(t)\}_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK^{\alpha} \mathbf{K}^{\beta} - C - \delta K,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$

Solving the representative agent's problem

Solution by the Pontryagin's maximum principle

- ▶ The current-value Hamiltonian

$$H(C, K, Q) = \frac{C^{1-\theta} - 1}{1-\theta} + Q \left(AK^\alpha \mathbf{K}^\beta - C - \delta K \right)$$

- ▶ The first order optimality conditions

$$\frac{\partial H}{\partial C} = 0 \iff C^{-\theta} = Q$$

$$\dot{Q} = \rho Q - \frac{\partial H}{\partial K} \iff \dot{Q} = (\rho + \delta - \alpha AK^{\alpha-1} \mathbf{K}^\beta) Q$$

$$\lim_{t \rightarrow \infty} K(t) Q(t) e^{-\rho t} = 0$$

- ▶ the admissibility conditions

$$\dot{K} = AK^\alpha \mathbf{K}^\beta - C - \delta K,$$

$$K(0) = k_0, \text{ given, } t = 0$$

Decentralized economy

Macroeconomic equilibrium

- ▶ first order optimality conditions for the agent

$$\dot{K} = AK^\alpha \mathbf{K}^\beta - C - \delta K, \quad (1)$$

$$\dot{C} = \frac{C}{\theta} \left(\alpha AK^{\alpha-1} \mathbf{K}^\beta - (\rho + \delta) \right), \quad (2)$$

$$K(0) = k(0) \quad (3)$$

$$0 = \lim_{t \rightarrow +\infty} C(t)^{-\theta} K(t) e^{-\rho t} \quad (4)$$

- ▶ macroeconomic consistency condition

$$K(t) = \mathbf{K}(t).$$

Decentralized economy

Macroeconomic equilibrium

- ▶ because we assumed that $\alpha + \beta = 1$ we get

$$\dot{K} = (A - \delta)K - C,$$

$$\dot{C} = \frac{C}{\theta} (\alpha A - (\rho + \delta)),$$

$$K(0) = k(0)$$

$$0 = \lim_{t \rightarrow +\infty} C(t)^{-\theta} K(t) e^{-\rho t}$$

- ▶ which is very similar to the AK model with the **exception of the rate of return for capital, $r = \alpha A$ instead of $r = A$**

Decentralized economy

Growth facts

- ▶ The rate of growth is

$$\bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- ▶ the consumption-capital ratio is

$$\frac{\bar{c}}{\bar{k}} = \beta_d,$$

for

$$\beta_d \equiv A - \delta - \bar{\gamma}_d = \frac{1}{\theta} (A(\theta - \alpha) + \rho + \delta(1 - \theta)) > 0$$

- ▶ the solution for output is

$$Y(t) = Ak_0 e^{\bar{\gamma}_d t}$$

Centralized economy

Centralized economy

Planner's problem

- ▶ The problem:
 - ▶ the planner has the **same utility** function as the agent (we have a representative agent economy)
 - ▶ but it considers the **total capital**, and not only the privately owned as the individual agent
 - ▶ that is, **externalities are internalized**. This means that the production function is

$$Y = AK^\alpha K^\beta = AK, \text{ if } \alpha + \beta = 1$$

Centralized economy

Planner's problem

- ▶ Formally: the planner's problem is

$$\max_{\{C(t)\}_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = AK - C - \delta K,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$
- ▶ It is indeed the *AK*-model

Comparison between the decentralized and centralized economies

- ▶ the rate of **growth in the centralized economy is higher than in the decentralised** economy

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > \bar{\gamma}_d = \frac{\alpha A - \delta - \rho}{\theta} > 0$$

- ▶ but **consumption is lower in the centralized economy than in the decentralised** economy

$$\left(\frac{C}{Y}\right)_c = \frac{A - \delta - \bar{\gamma}_c}{A} < \left(\frac{C}{Y}\right)_d = \frac{A - \delta - \bar{\gamma}_d}{A}$$

- ▶ rationale: in the decentralised economy the **incentive to saving is smaller** because the private rate of return of capital is also smaller

$$\text{(i.e., } r_c = \frac{\partial(AK)}{\partial K} = A > r_d = \frac{\partial(AK^\alpha \mathbf{K}^\beta)}{\partial K} = \alpha A)$$

Optimal government intervention

Decentralized economy with government intervention

Government intervention

- ▶ Assumption: there are two policy instruments - one distortionary (say taxes) and another non-distortionary (say expenditures)

$$T(t) = \tau Y(t), \quad G = \bar{G}(t)$$

- ▶ **two possible policies:**
distortionary taxes and lump sum transfers ($\tau > 0, G > 0$)
or
distortionary subsidies and lump sum taxes ($\tau < 0, G < 0$)
- ▶ assume the **government budget is balanced**

$$T(t) = G(t), \quad \forall t \in [0, \infty)$$

Decentralized economy with government intervention

Representative agent's problem

- ▶ the representative agent problem

$$\max_{[C(t)]_{t \geq 0}} \int_0^{\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to

$$\dot{K} = (1 - \tau)AK^\alpha \mathbf{K}^\beta - C - \delta K + G,$$

- ▶ boundary condition $\lim_{t \rightarrow \infty} k(t)h(t) \geq 0$

Decentralized economy with government intervention

Growth facts

- ▶ Using the same approach as before, the rate of growth is

$$\bar{\gamma}_g = \frac{\alpha(1 - \tau)A - \delta - \rho}{\theta} > 0$$

- ▶ can the rate of growth of the centralized economy be reached ?

$$\bar{\gamma}_c = \frac{A - \delta - \rho}{\theta} > 0$$

- ▶ **yes** if the tax rate is

$$\alpha(1 - \tau) = 1 \Rightarrow \tau = -\frac{1 - \alpha}{\alpha} < 0$$

- ▶ **Conclusion:** the government intervention can introduce incentives such that the externality is internalised by agents, in this case, by a **distortionary subsidy and a lump-sum tax** ($\tau > 0$ and $G > 0$)

References

- ▶ Romer (1986)
- ▶ (Acemoglu, 2009, ch. 11), (Aghion and Howitt, 2009, ch. 2) , (Barro and Sala-i-Martin, 2004, ch. 4)

Daron Acemoglu. *Introduction to Modern Economic Growth*.
Princeton University Press, 2009.

Philippe Aghion and Peter Howitt. *The Economics of Growth*.
MIT Press, 2009.

Robert J. Barro and Xavier Sala-i-Martin. *Economic Growth*.
MIT Press, 2nd edition, 2004.

Paul Romer. Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–37, October 1986.