A new-Keynesian economic growth model

Paulo B. Brito

18.4.2022

1 Introduction

In general two features distinguish new-Keynesian (NK) macroeconomics from Keynesian (K) and new-Classical (NC) macroeconomics:

1. unlike the first (K) but like the second (NC), it considers the micro-economic foundations of macroeconomics, by assuming that agents have a forward-looking behavior;

2. like the first (K) but unlike the second (NC), it assumes that there are market (or other) imperfections of any kind.

In dynamic macroeconomics, this implies that NK models are dynamic general equilibrium models (DGE) in which the equilibrium is usually not Pareto efficient.

Most NK models address business-cycle, i.e., short term issues related to the dynamics of inflation and to the design of monetary and fiscal policy to provide for macroeconomic adjustment.

As regards economic growth NK models have been developed to endogenize productivity growth by assuming that it is the result of R&D. As we will see in other lectures, R&D have some characteristics of a public good which entails the existence of externalities that are not internalized in a market economy, in the absence of policy interventions.

However, the non-existence of a competitive framework may also have growth consequences. The non-existence of a non-competitive environment can have several origins not necessarily related to the existence of a particular institutional framework.

Here we model a simple growth model with the following goals:

1. to see it as an extension of the AK model

2. to present a simple model for studying the effect of the degree of imperfect competition on growth.

2 The model

This section presents a new-Keynesian (NK) model for an economy with a structure similar to the AK and simple Romer model. It features a decentralized economy with two types of product markets, a final good and a continuum of intermediate goods, and one factor market (for capital). The final good market and the factor market are competitive (in the sense that all participants are price-takers) but there is monopolistic competition in the intermediate goods' markets.

We will see that the equilibrium representation is similar to the Romer externality model, with the difference that there is a markup over the marginal rate of return of capital.

In this section we present the problems of the three types of representative agents of the model: households, the final good producer and the generic problem for an intermediate good producer. The last subsection defines the DGE for this economy.

We introduce several simplifying assumptions. In particular, we assume that the final product sector only uses intermediate inputs, all physical capital is used in the intermediate goods' sector, and there is a classic dichotomy between prices and quantities. This means, in the NK terminology, that there are no real or nominal rigidities.

2.1 The household problem

The representative household consumes, saves and invests in a risk-free financial asset. It has an initial level of net financial wealth, and receives a flow of financial income. We assume that its preferences are represented by an intertemporal additive utility functional and a CRRA utility function. Its problem is

$$
\begin{aligned}
\max_C V[C] = \int_0^\infty & \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t}\, dt \\
& \text{subject to} \\
& \dot{W} = r_w(t) W(t) - C(t) \\
& W(0) = W_0, \text{ given} \\
& \lim_{t\to\infty} e^{-R_w(t)} W(t) \geq 0,
\end{aligned}
\tag{1}
$$

where $C(\cdot)$, and $W(\cdot)$ denote real consumption, and the real net financial wealth. In addition, $r_w(t)$ denotes the real interest rate and $R(t) = \int_0^t r(s)ds$ is the return at time t. Observe that, because the rate of return of capital is endogenous at the general equilibrium level, we set is a as a function of time. All the variables are denoted in real terms, deflated by the final good price $P(t) = 1$.

The first order conditions, are, from Pontryiagin's maximum principle

$$\dot{W} = r_w(t)W(t) - C(t) \tag{2}$$

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r_w(t) - \rho) \tag{3}$$

together with the initial condition $W(0) = W_0$ and the transversality condition $\lim_{t\to\infty} C(t)^{-\theta} W(t) e^{-\rho t} = 0$.

2.2 Final producer problem

Final output uses a continuum of intermediate goods of varieties $j \in [0,1]$ with a constant elasticity of substitution (CES) technology, via a Dixit and Stiglitz (1977) aggregator. Denoting the quantity of intermediate input of variety j used at time t by $x(j,t)$ and by $\mathbf{x}(t) = \big(x(j,t)\big)_{j\in[0,1]}$ the ensemble of all inputs the production function is a functional over $x(\cdot)$,

$$F\big(\mathbf{x}(t)\big) = \left(\int_0^1 x(j,t)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where $\varepsilon > 1$ is the elasticity of substitution between. Denoting $\mu \equiv 1/\varepsilon \in (0,1)$ the Lerner index, which is a measure of the degree of market power, we see that it increases if the elasticity is smaller. This mea

The profit can be written as a functional over $\mathbf{x}$,

$$\pi(\mathbf{x}(t)) = P(t)\,Y(t) - TC(\mathbf{x}(t)),$$

where $TC(\mathbf{x}) = \int_0^1 p(j,t)\,x(j,t)\,dj$ is total cost (again a functional over $x(\cdot)$).

The final producer is a price-taker in both markets, inputs and output, seeks to maximize the total costs for producing the output quantity $Y(t)$, where given the ensemble of input prices $\mathbf{p} = \big(p(j,t)\big)_{j\in[0,1]}$

$$\max_{x(\cdot,t)} \pi(\mathbf{x}(t)) \text{ s.t. } F\big(\mathbf{x}(t)\big) = Y(t) \tag{5}$$

The demand for input j is[1]

$$x^d(j,t) = \left(\frac{p(j,t)}{P(t)}\right)^{-\varepsilon} Y(t), \text{ for every } j \in [0,1], \text{ and } t \in [0,\infty) \tag{6}$$

[1]See Appendix A.

where because of perfect competition the market price is equal to the marginal cost of producing one unit of output

$$P(t) = P^*(t) = \left(\int_0^1 p(j,t)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \quad t \in [0, \infty).$$

Because we have no monetary variables we will set $P(t) = 1$, for all t. We see that the demand of variety j is negatively related with its own price and positively related with the output product price and the level of activity Y.

2.3 Intermediate producer of variety j

We assume that the number of industries, which produce intermediate inputs, is fixed and is normalized to one.

Next we present and solve the problem for the firm in any of the industry $j \in [0,1]$, assuming that this producer is a monopolist. This is a case of monopolistic competition (MC) because, although the firm in any industry is a price-setter on the market for its output, $p(j,\cdot)$, it has to compete with all other industries in the supply of intermediate products to the producer of the final good, whose demand function is given in equation (6). An increase in the price of product j will generate a reduction in its demand.

The technology for product j is benchmark of the AK type

$$x(j,t) = A(j)\, k(j,t) \tag{7}$$

where $k(j,t)$ is the capital and labour inputs for producing $x(j,t)$.

In order to simplify the model we assume a homogeneous technology across industries. In particular, it is assumed that the TFP is not product specific, that is $A(j) = A$ for every $j \in [0,1]$.

The instantaneous profit for producer j is, in real terms evaluated at the price of the final good

$$\pi(j,t) = \frac{x(j,t)p(j,t)}{P(t)}$$

where $x(j,t)$ is equal to the demand for intermediate product j by the final producer, using the price of the final product as a numéraire.

Substituting the demand function (6) yields

$$\begin{aligned}\pi(j,t) &= \frac{x(j,t)p(j,t)}{P(t)} = \\ &= \left(\frac{x(j,t)}{Y(t)}\right)^{-\frac{1}{\varepsilon}} x(j,t) = \\ &= x(j,t)^{1-\mu} Y(t)^{\mu}\end{aligned}$$

where $\mu \equiv 1/\varepsilon \in (0,1)$ is the Lerner index.

If we substitute the production function (7), the profit of the intermediate producer j becomes

$$\pi(j,t) = \Big(A\,k(j,t)\Big)^{1-\mu} Y(t)^{\mu}. \tag{8}$$

As we assume that capital is used in the production of intermediate goods, the problem of each intermediate producer is dynamic. We assume the classic Jorgenson (1967) model for the producer in which there are no adjustment costs in investment.

The firm's objective is to maximize the present-value of the cash-flow discounted by the market interest rate, subject to the accumulation equation for capital:

$$\begin{aligned}&\max_{i(j,\cdot)} \int_0^{\infty} \left(\pi(j,t) - i(j,t)\right) e^{-R(t)}\, dt \\ &\quad \text{subject to} \\ &\quad \frac{dk(j,t)}{dt} = i(j,t) - \delta k(j,t), \text{ for each } t \in [0,\infty) \\ &\quad k(j,0) = k_{j,0}, \text{ given}\end{aligned} \tag{9}$$

for $R(t) = \int_0^t r(s)\, ds$, where $r(t)$ is the market rate of return of capital, and we assume that $k(j,t)$ is asymptotically non-negative. The optimum demand for labor and capital are symmetric, in the sense that they are the same for producer of all intermediate goods,[2]

$$k(j,t) = \left(\frac{1-\mu}{r(t)+\delta}\right) \pi(j,t), \text{ for each } j \in [0,1]. \tag{10}$$

In Appendix B we find the profit at the optimum

$$\pi^*(j,t) = \pi^*(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1-\mu}{\mu}} Y(t), \text{ for each } j \in [0,1]$$

[2]See Appendix B.

which is symmetric across varieties. The optimal profit level, which is a function of the cost of capital inputs and of the output of the final good. Differently from the final producer case, the profit is different from zero because there is monopolistic competition (MC): the intermediate producer of variety j is a monopolist in its own market but competes with the producer of all the other varieties that enter in the production of the final output $Y(t)$. It has a Lerner-markup given by $1-\mu = \frac{\varepsilon-1}{\varepsilon}$ which decreases with the elasticity of substitution ε.

From equation (**??**) we find the capital stock allocated to variety j

$$k^*(j,t) = k^*(t) = \Big(\frac{(1-\mu)\,A}{r(t)+\delta}\Big)^{\frac{1}{\mu}} \frac{Y(t)}{A}, \text{ for each } j \in [0,1]. \tag{11}$$

Using the production function yields the optimal supply of intermediate inputs, that is also symmetric across industries,

$$x^*(j,t) = x^*(t) = \Big(\frac{(1-\mu)\,A}{r(t)+\delta}\Big)^{\frac{1}{\mu}} Y(t), \text{ for each } j \in [0,1]. \tag{12}$$

2.4 Aggregation

Because different inputs are measured in real terms, they can be measured in different physical units. Therefore, in order to build the aggregate capital stock and labor input we need to choose an appropriate aggregator. If we use the Dixit and Stiglitz (1977) aggregator we have the aggregate demand for capital and labor

$$K(t) = \Big(\int_0^1 k^*(j,t)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\Big)^{\frac{\varepsilon}{\varepsilon-1}} = k^*(t)$$

where $k^*(t)$ is given in equation (11). Total investment is also obtained from

$$I(t) = \Big(\int_0^1 i^*(j,t)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\Big)^{\frac{\varepsilon}{\varepsilon-1}}$$

although it is indeterminate, as is well known in a Jorgenson (1967) model, but can be determined at the equilibrium level.

The supply of the final good can be determined substituting $x^*(j,t)$, from equation (12) in the production function (4)

$$
\begin{aligned}
Y(t) &= \Big(\int_0^1 x^*(j,t)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \Big)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \Big(\int_0^1 x^*(t)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \Big)^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \Big(\frac{(1-\mu)\, A}{r(t)+\delta} \Big)^{\frac{1}{\mu}} Y(t).
\end{aligned}
$$

This has two implications: first, the cost of capital satisfies

$$
r(t) + \delta = (1-\mu)\, A, \text{ for every } t \in [0,\infty), \tag{13}
$$

and the aggregate output from the intermediary sector is

$$
X(t) = \Big(\int_0^1 x^*(j,t)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \Big)^{\frac{\varepsilon}{\varepsilon-1}} = Y(t).
$$

2.5 General equilibrium

The **general equilibrium** is defined by the aggregate flows of consumption, $\big(C(t)\big)_{t\in[0,\infty)}$, output $\big(Y(t)\big)_{t\in[0,\infty)}$, and by the allocation flows of production, capital input, labor input and prices of intermediate-goods, $\Big(\big(x(j,t), k(j,t), p(j,t)\big)_{j\in[0,1]}\Big)_{t\in[0,\infty)}$, the rate of return of capital $\big(r(t)\big)_{t\in[0,\infty)}$, and the wage rate $\big(\omega(t)\big)_{t\in[0,\infty)}$ such that

1. households solve their problem, in equation (1), given the rate of return and the wage rate;
2. the final producer solve its problem, in equation (5), given the prices of the intermediate goods;
3. every intermediate producer, $j \in [0,1]$, solve its problem, in equation (9), given the rate of return and the wage rate;
4. the balance sheet and consistency conditions hold: $W(t) = K(t)$ (households own firms);
5. the market clearing conditions:
    - for the final good market

$$
Y(t) = C(t) + I(t), \text{ for each } t \in [0,\infty)
$$

- for the j intermediary goods

$$x^d(j,t) = x^*(j,t), \text{ for each } (j,t) \in [0,1] \times [0,\infty),$$

- for the capital market

$$r_w(t) = r(t), \text{ for each } t \in [0,\infty).$$

3 General equilibrium representation and dynamics

The equilibrium condition in the capital market and equation (13) yield

$$r_w(t) = (1-\mu)\,A - \delta.$$

Therefore, in equilibrium, the Euler equation becomes

$$\dot{C} = C(t)\left(\frac{(1-\mu)\,A - \delta - \rho}{\theta}\right)$$

and, using the balance-sheet condition, the budget constraint of the household $\dot{W} = r_w(t)\,W(t) - C(t)$, becomes

$$\dot{K} = (1-\mu)\,A\,K(t) - \delta\,K(t) - C(t). \tag{14}$$

The equilibrium condition in market for variety j is

$$\left(\frac{p(j,t)}{P(t)}\right)^{\frac{1}{\mu}} Y(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1}{\mu}} Y(t) = Y(t)$$

and therefore $p(j,t) = P(t) = 1$ for every (j,t).

At last, as

$$Y(t) = X(t) = A\,K(t)$$

and the equilibrium in the market for the final good condition, $Y(t) = C(t) + I(t)$ together with equation (14) yields the investment expenditure in equilibrium,

$$I(t) = A\,K(t) - C(t) = \dot{K} + \big(\mu\,A + \delta\big)\,K(t),$$

or, equivalently, the relationship between net investment and investment expenditures

$$\dot{K} = I(t) - (\mu\,A + \delta)\,K(t).$$

This means that the existence of imperfect competition entails a kind of adjustment cost.

3.1 Equilibrium characterization

Therefore the equilibrium is the solution to the system

$$
\begin{aligned}
\dot{K} &= (1-\mu)\, A\, K(t) - C(t) - \delta K(t) \\
\dot{C} &= C(t) \frac{(1-\mu)\, A - \rho - \delta}{\theta} \\
K(0) &= k_0 \text{ given} \\
\lim_{t\to\infty} & K(t)\, C(t)^{-\theta}\, e^{-\rho\, t} = 0.
\end{aligned}
$$

which lis similar to the one we found for the AK model, with a distortion introduced by $m \equiv 1-\mu$ which is an (exogenous) markup.

3.2 Growth implications

Given the linear structure, we can decompose the both variables as

$$K(t) = k_d(t)\, e^{\gamma\, t} \text{ and } C(t) = c_d(t)\, e^{\gamma\, t}.$$

We obtain a dynamic system for the detrended variables

$$
\begin{aligned}
\dot{k}_d &= \big((1-\mu)\, A - \delta - \gamma\big)\, k_d(t) - c_d(t) \\
\dot{c}_d &= c_d \Big(\frac{(1-\mu)\, A - \rho - \delta}{\theta} - \gamma \Big).
\end{aligned}
$$

At the steady state of this system we obtain the long-run growth rate

$$\bar{\gamma} = \frac{(1-\mu)\, A - \rho - \delta}{\theta} \tag{15}$$

and the consumption-capital ratio

$$\beta = \frac{\big((1-\mu)\, A - \delta\big)\, (\theta - 1) + \rho}{\theta}$$

that should be positive for the verification of the transversality condition (prove this).

Recall that in a model with perfect competition, the AK model we obtained

$$\gamma_{AK} = \frac{A - \rho - \delta}{\theta}$$

and

$$\beta_{AK} = \frac{\big(A - \delta\big)\,(\theta - 1) + \rho}{\theta}.$$

Therefore output of the final good is

$$Y(t) = A\,k_0\,e^{\bar{\gamma}\,t},\ \text{for}\, t \in [0, \infty)$$

which allows us to derive the following growth facts regarding the effects of imperfect competition on growth:

1. the rate of long run growth is smaller the higher is the deviation from perfect competition, i.e.

$$\frac{\partial \bar{\gamma}}{\partial \mu} = -\frac{A}{\theta} < 0$$

   or, in other words, the long-run growth rate is lower than in the AK model.

2. The level of the GDP along the BGP is not affected by imperfect competition. However, the consumption level will be smaller than in the AK model

$$c_d = \beta\,k_0 < c_{AK} = \beta_{AK}\,k_0$$

3. The solution is along a BGP. That is, there is no transition dynamics.

References

Dixit, A. and Stiglitz, J. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, 67(3):297–308.

Jorgenson, D. (1967). The theory of investment behavior. In Ferber, R., editor, *Determinants of Investment Behavior*, pages 129–175. NBER.

A Solution of final producer problem (5)

The Lagrangean is, ignoring the time argument,

$$L(\mathbf{x}, \lambda) = P\,Y - TC(\mathbf{x}) + \lambda\,\big(F(\mathbf{x}) - Y\big)$$

where λ is the Lagrange multiplier (an unknown constant). The first order conditions are

$$\begin{cases} \dfrac{\delta L(\mathbf{x})}{\delta x(j)} = 0 & \iff \quad \dfrac{\delta TC(\mathbf{x})}{\delta x(j)} = \lambda \dfrac{\delta F(\mathbf{x})}{\delta x(j)} \iff p(j) = \lambda \left(\dfrac{F(\mathbf{x})}{x(j)}\right)^{\frac{1}{\varepsilon}}, \text{ for every } j \in [0,1] \\ \dfrac{\partial L(\mathbf{x})}{\partial \lambda} = 0 & \iff \quad F(\mathbf{x}) = Y. \end{cases}$$

Then $\lambda^{\varepsilon}\, Y = p(j)^{\varepsilon}\, x(j)$ for any $j \in [0,1]$. Then, the demand for input j is

$$x(j) = \lambda^{\varepsilon} p(j)^{-\varepsilon}\, Y,$$

depends on the Lagrange multiplier. But substituting in the constraint,

$$F(\mathbf{x}) = \left(\int_0^1 \left(\lambda^{\varepsilon} p(j)^{-\varepsilon}\, Y\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = Y$$

and solving for λ, we find that

$$\lambda^* = \left(\int_0^1 p(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} = P^*$$

is the shadow cost of one unit of output. At the optimum, the total cost is

$$\begin{aligned} TC^* &= \int_0^1 p(j)\, x(j)\, dj = \\ &= \int_0^1 \lambda^{\varepsilon}\, p(j,t)^{1-\varepsilon}\, Y\, dj = \\ &= \lambda^{\varepsilon}\, Y \int_0^1 p(j)^{1-\varepsilon}\, dj = \\ &= \lambda^{\varepsilon}\, \lambda^{1-\varepsilon}\, Y \\ &= P^*\, Y. \end{aligned}$$

Substituting in the profit functional yields

$$\pi^*(t) = Y(t)\left(P(t) - P^*(t)\right).$$

As the firm is price taker then $P^*(t) = P(t)$ and there is zero profit for every t. Therefore, the demand function is given in equation (6).

B Solution of the intermediate producer j problem (9)

The current-value Hamiltonian is

$$H(j,t) = \pi(j,t) - i(j,t) + q(j,t)\left(i(j,t) - \delta k(j,t)\right).$$

where $\pi(j,t) = \Big(A\,k(j,t)\Big)^{1-\mu} Y(t)^{\mu}$ is the real profit for the producer of variety j.

The static optimality condition is

$$\frac{\partial H(j,t)}{\partial i(j,t)} = -1 + q(j,t) = 0 \tag{16}$$

and the Euler equation and the transversality conditions are

$$\begin{aligned}\frac{dq(j,t)}{dt} &= r(t)q(j,t) - \frac{\partial H(t)}{\partial k(j,t)} \\ &= \left(r(t)+\delta\right)q(j,t) - (1-\mu)\,\frac{\pi^*(j,t)}{k(j,t)}\end{aligned} \tag{17}$$

and $\lim_{t\to\infty} e^{-R(t)}\,q(j,t)\,k(j,t) = 0$ for all j.

Equation (16) yields

$$q(j,t) = q(t) = 1, \text{ for every } j \in [0,1] \text{ and } t \in [0,\infty),$$

that is the shadow value of capital is the same for all sectors $q(t) = q(j,t)$ and it is constant $q(t) = 1$. Therefore, the gross rate of return of capital is obtained from (17)

$$r(t) + \delta = (1-\mu)\,\frac{\pi^*(j,t)}{k^*(j,t)}$$

Substituting in the profit function

$$\begin{aligned}\pi^*(j,t) &= \Big(A\,k^*(j,t)\Big)^{1-\mu} Y(t)^{\mu} \\ &= \left(A\,(1-\mu)\,\frac{\pi^*(j,t)}{r(t)+\delta}\right)^{1-\mu} Y(t)^{\mu}\end{aligned}$$

yields

$$\pi^*(j,t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1-\mu}{\mu}} Y(t),$$

which means that profits are symmetric across variety producers. Therefore, factor demand function is symmetric as well

$$k^*(j,t) = k^*(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1}{\mu}} \frac{Y(t)}{A}.$$

Therefore there is also symmetry in the supply of varieties,

$$x^*(j,t) = x^*(t) = \left(\frac{(1-\mu)\,A}{r(t)+\delta}\right)^{\frac{1}{\mu}} Y(t) \text{ for any } j \in [0,1], \text{ for each } t \in [0,\infty).$$

The total cost to the firm,

$$(r(t)+\delta)\,k^*(j,t) = (1-\mu)\,\pi^*(j,t),$$

is distributed to the household.