

Closed book exam. No auxiliary material ( on paper, electronic or any other form) is allowed.

1. [5 points] Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is  $\{y_0, y_1\}$  and the intertemporal utility function is

$$U(c_0, c_1) = \log c_0 + \beta \log c_1, \quad 0 < \beta < 1$$

- a) Specify the agent's problem. Define the general equilibrium.  
b) Characterize the implicit behavioral assumptions. Solve the representative agent problem.  
c) Find the equilibrium asset return. Provide an intuition.
2. [7 points] Consider Arrow-Debreu economies with the data that follows: (1) the information is given by a binomial tree with two periods,  $\mathbb{T} = \{0, 1\}$  and  $N$  states of nature for period 1; (2) the endowment distribution for the period  $t = 1$  is  $y_{1,s} = y_0 \cdot (1 + \gamma_s)$ , for state  $s = 1, \dots, N$ ; (3) agents are homogenous; (4) the representative agent has a discounted time-additive, von-Neumann-Morgenstern utility functional with a CARA Bernoulli utility function,

$$u(c) = \frac{c^{1-\theta}}{1-\theta}, \quad \theta > 0$$

- (a) Define the equilibrium, and provide an intuition.  
(b) Determine the solution for the consumer problem, and provide an intuition.  
(c) Determine the equilibrium stochastic discount factor. Assuming that  $\mathbb{E}[\gamma_s] = \gamma > 0$  find a bound to the expected value of the stochastic discount factor by using Jensen's inequality. Provide an intuition for your results.
3. [8 points] Consider a finance economy in which there is a risk-free asset with return  $1 + r$ , for  $r > 0$ , and a risky asset with return  $R = (1 + r - \varepsilon, 1 + r + \varepsilon)$  for  $0 < \varepsilon < 1$ . Assume that the representative consumer has a logarithmic Bernoulli utility function and the endowment process is  $Y = \{1, Y_1\}$  where  $Y_1 = (1 - \gamma, 1 + \gamma)$  for  $0 < \gamma < 1$ .
- (a) Find the probabilities for the two states of nature assuming that there are no arbitrage opportunities (Hint: use the condition  $\mathbb{E}[M(R - R^f)] = 0$ ).  
(b) Find the Sharpe index for the risk premium. Justify.  
(c) Find the Hansen-Jaganathan bound. Justify.  
(d) Does the equity premium puzzle holds in this case ? Provide an intuition.