Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Mestrado em Economia Monetária e Financeira
Fundamentos de Economia Financeira
2017-2018

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Prova de avaliação: **Época Recurso**
3.7.2018 (18.00h-21.00h, 113 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. `[6 points]` Consider two alternative investment projects, labelled A and B, with possibly contingent profits. The profit of project $i \in \{A, B\}$, in state of nature $s \in \{1, 2\}$, is $\Pi^i_s = p^i y^i_s - c^i$, where p^i is the selling price, y^i_s is the contingent output, and c^i is the cost. Consider the data for the two projects given in the following table:

Investments		$i = A$	$i = B$
selling price	p^i	$1 + \phi$	$1 + \phi$
cost	c^i	1	1
output in state $s = 1$	y^i_1	$1 - \gamma$	1
output in state $s = 2$	y^i_2	$1 + \gamma$	1

Furthermore, assume that the two states of nature have equal probabilities, and that $0 < \gamma < 1$ and $\phi > 0$. The value of project i be determined by $V^i = \mathbb{E}[u(\Pi^i)]$, where $u(.)$ is a Bernoulli utility function, to be specified next.

(a) Assume that the agent has the utility function $u(\Pi) = \Pi$. How would the investor rank the projects ?

(b) Now assume that the agent values the projects with the utility function $u(\Pi) = \Pi - \frac{1}{2}\,\Pi^2$. How would the investor rank the projects in this case?

(c) Provide an intuition for the results you obtained in (a) and (b).

2. `[7 points]` Consider an endowment finance economy where the asset market is characterized by the vector of asset prices and the payoff matrix

$$\mathbf{S} = \begin{pmatrix} \frac{1}{1+i} & s \end{pmatrix} \text{ and } \mathbf{V} = \begin{pmatrix} 1 & v_1 \\ 1 & v_2 \end{pmatrix}$$

where $v_1 < 1 < v_2$ and $s > 0$. Agents are homogeneous and the endowment process is $\{y_0, Y_1\} = \{1, (1+\gamma_1, 1+\gamma_2)\}$, for arbitrary values of γ_1 and γ_2. They value the consumption process $\{ c_0, C_1\}$ by a von-Neumann-Morgenstern utility functional with discount factor β and a Bernoulli utility function $u(c) = \dfrac{c^{1-\theta}}{1-\theta}$.

(a) Define the general equilibrium for this economy

(b) Solve the representative agent problem.

(c) Find the equilibrium returns for the two assets.

3. `[7 points]` Consider a finance economy in which the information tree is binomial and in which there are three assets with the vector of prices and payoff matrix given by

$$\mathbf{S} = \begin{pmatrix} s & 1 & \frac{1}{1+r} \end{pmatrix} \quad \mathbf{V} = \begin{pmatrix} s \times (1 + r + \epsilon) & 1 + r - \epsilon & 1 \\ s \times (1 + r - \epsilon) & 1 + r + \epsilon & 1 \end{pmatrix}$$

where we assume that $r > 0$, $s > 0$, and $0 < \epsilon < 1$.

(a) Determine the state prices and characterize the finance market regarding the existence of arbitrage opportunities and completeness.

(b) Consider the introduction of an European call option, over the first risky asset, with exercise payoff $s \times (1+r)$. What should be its price ? What should be the composition of the replicating porfolio ?

(c) Assume that the probability of the first state of nature is higher than $\frac{1}{2}$. Compute the Sharpe index for the second asset (i.e, with price $S^2 = 1$ and payoff $V^2 = (1+r-\epsilon, 1+r+\epsilon)^\top$. Explain

(d) Assume a general equilibrium finance endowment economy in which the representantive consumer has a von-Neumann-Morgenstern utility functional, and a logarithmic Bernoulli utility function. Assume that the previous assumption on the probabilities of the states of nature holds, and the endowment has a growth factor given by the process $1+\Gamma$. Find the (general) equilibrium return for the risk-free asset and compute an upper estimate for the risk free interest rate.