Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Master in Monetary and Financial Economics
Fundamentals of Financial Economics
2019-2020

Lecturer: Paulo Brito
Exam: **Época Normal** (First exam)
8.6.2020
First part: 18:00 - 19:00

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
    1. In your answer to questions involving analytical derivations, please start with a **very short explanation** of your reasoning.
    2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Points: 1(a) - 1, 1(b) - 1.5, 1(c) - 1.5, 2(a) - 3, 2(b) - 2, and 2(c) - 1.
- Your exam will only be considered if it is uploaded in Aquila between 19:00 and 19:05.

1 Consider a two-period finance economy in which the information is given by a binomial tree with objective probabilities (π_1, π_2). The financial market is characterized by the return matrix in the (state $\times$ asset) form

$$\begin{pmatrix} 1 & R_1 \\ 1 & R_2 \end{pmatrix}.$$

(a) Provide conditions for the absence of arbitrage opportunities and for the existence of complete markets. Consider those conditions from now on.

(b) Deduce the Sharpe index (tip: start by proving that the covariance between the two random variables $X = (x_1, x_2)$ and $Y = (y_1, y_2)$, adapted to the previous binomial tree, is $COV(X, Y) = \pi_1 \pi_2 (x_1 - x_2)(y_1 - y_2)$). What are the consequences of the conditions you have derived in (a) on the sign and magnitude of the Sharpe index.

(c) Find a relationship between the objective and the risk-neutral probabilities such that the expected risk premium is non-negative. Explain.

2 Consider the same information tree and the same financial markets as in question 1. Recall, in particular, the conditions you introduced for the existence of absence of arbitrage conditions and complete markets. In this economy there is a representative household who solves the problem

$$\max_{c_0, C_1, \ell, \theta} u(c_0) + \beta \, \mathbb{E}[u(C_1)], \; 0 < \beta < 1$$

subject to the budget constraint $c_0 = y_0 - \ell - \theta$ at time $t = 0$ and $c_{1,s} = \ell + \theta \, R_s + y_{1s}$ at time $t = 1$, for the states of nature $s = 1, 2$. We denote by $\{c_0, C_1\}$ and $\{y_0, Y_1\}$, where $y_{11} \neq y_{12}$, the processes for consumption and (exogenous) endowments, and ℓ and θ the (long) positions on money and the risky asset. Assume that the utility function is $u(c) = \ln(c)$.

(a) Solve the household problem.

(b) Derive the conditions for the existence of full insurance at the household level. Under the previous conditions, find the relationship between the optimal household position in the risky asset and the covariance between the return of the risky asst and the endowment at time $t = 1$, i.e., $COV(R, Y_1)$. Provide an intuition for the two results.

(c) Assume this is a homogeneous agent economy. Is it possible to have complete insurance at the general equilibrium level ? Provide an explanation.