

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
 1. Please provide a **very short explanation** of your reasoning. In its absence, your response, even if correct, may be discounted.
 2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Points: 1(a) - 1, 1(b) - 1, 1(c) - 2, 2(a) - 2, 2(b) - 3, and 2(c) - 1.
- Your exam will only be considered if it is uploaded in Aquila between 19:00 and 19:05.

1 Consider a two-period intertemporal utility function, in a deterministic setting, for the consumption sequence $\{c_0, c_1\}$

$$U(c_0, c_1) = (1 - \beta) \ln(c_0) + \beta \ln(c_1), \text{ with } 0 < \beta < 1.$$

- (a) After determining the intertemporal marginal rate of substitution, the Allen-Uzawa elasticities, and the elasticity of intertemporal substitution, characterize the possible types of household behavior through time.
- (b) Assume a representative-agent Arrow-Debreu (AD) endowment economy for which the flow of endowment is $\{y_0, (1 + \gamma)y_0\}$. Find the general equilibrium. Characterize the equilibrium state-price under the existence of impatience. Under which conditions equilibrium consumption will be constant through time ?
- (c) Assume instead that there are two groups of agents a and b , which are heterogeneous only as regards their sequence of endowments. Those agents have the following flow of endowments, $y^a = \{y_0, 0\}$ and $y^b = \{0, (1 + \gamma)y_0\}$, with $y_0 > 0$ and $\gamma > 0$, respectively. Find the general equilibrium in this economy and compare with the equilibrium for a representative agent economy in (b). Provide a brief discussion on the difference between the two equilibria.

2 Assume a representative-agent Arrow-Debreu (AD) endowment economy, in a stochastic environment, where the flow of endowments is $\{y_0, (\mathbf{1} + \Gamma)y_0\}$ where $\mathbf{1} + \Gamma = (1 + \gamma_1, \dots, 1 + \gamma_s, \dots, 1 + \gamma_n)$.

- (a) Find the dynamic stochastic general equilibrium, assuming that the representative consumer has the intertemporal utility functional

$$U(c_0, c_1) = (1 - \beta) \ln(c_0) + \beta \mathbb{E}[\ln(C_1)], \text{ with } 0 < \beta < 1,$$

over the consumption sequence $\{c_0, C_1\}$, where $C_1 = (c_{11}, \dots, c_{1s}, \dots, c_{1n})$,

- (b) Find the dynamic stochastic general equilibrium, assuming instead that the representative consumer has the intertemporal utility functional

$$U(c_0, c_1) = (1 - \beta) \ln(c_0) + \beta \ln(CE[C_1]), \text{ with } 0 < \beta < 1.$$

where $CE[C_1]$ is the certainty equivalent associated to the utility function $u(c_{1s}) = \frac{c_{1s}^{1-\sigma} - 1}{1-\sigma}$, with $\sigma \geq 0$.

- (c) Compare the equilibrium stochastic discount factor (ESDF) you have derived in (a) with the one you have derived in (b), addressing specifically the cases in which $\sigma = 0$ and $\sigma > 0$. Provide an intuition.