

Warning:

- This is an online open book exam. This implies that in the assessment the following two points will be taken into consideration:
 1. Please provide a **very short explanation** of your reasoning. In its absence, your response, even if correct, may be discounted.
 2. Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- **Points:** 1(a) - 1, 1(b) - 1.5, 1(c) - 1.5, 2(a) - 2, 2(b) - 2, and 2(c) - 2.
- Your exam will only be considered if it is uploaded in Aquila between 19:30 and 20:10.

- 1 Consider an economic policy authority (EPA) in charge of assessing and controlling the economic growth of an economy, for the period of one year. It has the following information: it observes the growth factor $g_0 = 1 + \gamma$, at the beginning of the year, and it assumes that the growth factor follows a binomial random variable $G_1 = (g_1, g_2) = (1 + \gamma - \sigma, 1 + \gamma + \sigma)$, for $0 < \sigma < 1 + \gamma$, at the end of the year.
- (a) If the EPA assumes that the process $\{g_0, G_1\}$ is a martingale (tip: a martingale is a process $\{X_t\}_{t=0}^T$ such that $\mathbb{E}_t[X_{t+1}] = x_t$), what will be the expected value and the standard deviation for G_1 ?
- (b) The EPA measures the cost of macroeconomic volatility by $C(G_1) = \mathbb{E}[G_1] - CE(G_1)$, where $CE(G_1)$ is the certainty equivalent of the growth factor, assuming an utility function $u(g) = \ln(g)$. Find $C(G_1)$. Explain its meaning.
- (c) Let the EPA have a state-independent instrument $\tau \in (-g_0, g_0)$ that can additively change the growth factor to $\tilde{G}_1(\tau) = (g_1 + \tau, g_2 + \tau)$. If the EPA would use the instrument τ in order minimize $\tilde{G}_1(\tau)$ what would be the minimum cost of volatility that it can achieve ? Can it be zero ? Why ?
- 2 Consider an homogeneous agent endowment finance economy in which there is money, with a return equal to 1 at all times and states of nature, and a risky asset with return $R = (1 + \varepsilon, 1 - \varepsilon)$, for $\varepsilon > 0$. The endowment process is $Y = \{y_0, Y_1\}$ where $Y_1 = ((1 + \gamma)y_0, (1 - \gamma)y_0)$ for $0 < \gamma < 1$. The representative consumer has the intertemporal utility functional

$$U(c_0, C_1) = \frac{c_0^{1-\theta} - 1}{1-\theta} + \beta \sum_{s=1}^2 \pi_s \frac{c_{1s}^{1-\theta} - 1}{1-\theta},$$

for $0 < \beta < 1$ and $\theta > 1$. Assume that all those parameters, $(\varepsilon, \gamma, \beta, \theta)$, in this economy are known.

- (a) Find the dynamic stochastic general equilibrium for this economy.
- (b) Find the probabilities for the two states of nature assuming that there are no arbitrage opportunities (tip: use the condition $\mathbb{E}[M(R - R^f)] = 0$ where R^f is the return for the risk-free asset and M is the equilibrium stochastic discount factor.).
- (c) Find the Sharpe index and the Hansen-Jaganathan bound. Provide an intuition for your results.