Foundations of Financial Economics
Financial frictions: moral hazard

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April 23, 2021

This lecture

- General equilibrium with moral hazard: the Holmstrom Tirole model
- We consider again the ”internal” finance model: demand and supply of funds between heterogeneous agents
- Main difference from the benchmark model: **asymmetric information**
- In this case, we consider **moral hazard** (or the principal-agent model): one party does not observe the **actions** of the other
- This generates a **financial friction**: a borrowing constraint
- And a balance effect: **the distribution of wealth between agents has an effect on the interest rate**
- This provides a solid theoretical underpinning to a old theory of interest rates: the loanable fund theory.

Topics

- The lender's problem
- Contracts in the presence of moral hazard
- Financial friction: borrowing constraint
- The borrower's problem
- Equilibrium interest rate.
- **Simplifying assumption**: the resources of the economy take the form of financial wealth distributed at the at the beginning of period 0.

The lender

The lender's problem

Assumptions

- Has liquid net worth W^l, that is higher than the desired consumption at time $t = 0$, and its the only way to finance consumption at time $t = 1$.
- **Lends θ^l through a debt contract** in which the return at time $t = 1$ is **risk-free**. Therefore consumption at time $t = 1$ is risk free.
- The **lender's problem** is

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \theta^l = W^l, \; c_1^l = R\theta^l$$

  where R is the return on the asset.
- The Bernoulli utility function is concave: $u^{''}(c) < 0 < u^{'}(c)$

The lender's problem

Solution

- Equivalently

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \frac{c_1^l}{R} = W^l$$

- Assuming a log utility function the solution is

$$c_0^l = \frac{1}{1+\beta} W^l, \; c_1^l = \frac{\beta R}{1+\beta} W^l$$

- The demand for the asset, or the **liquidity supply**, is

$$\boxed{\theta^l = \frac{c_1^l}{R} = \frac{\beta}{1+\beta} W^l}$$

The borrower

The borrowers's project
Assumptions

- Has net worth W^b
- Wants to invest I in a project. If $I \geq W^b$ needs to borrow $\theta^b = W^b - I < 0$ from the lender.
- But the net payoff of the project depends from the borrowers' actions (which are random from the perspective of the lender);
- The borrower can follow one of the two courses of action (**not observable by the lender**):
  - put high effort and use all the resources in the project
  - put low effort and divert resources from the project (or having a more inefficient management)
- The probability of success depends on the effort level ($p_H > p_L$).

The borrowers's project

Expected returns

- The expected returns, obtained at period $t = 1$ from the courses of action are: with expected returns
  - Good project: $E[V_H] = p_H \frac{V}{p_H} + (1 - p_H)0 = V$
  - Bad project: $E[V_L] = p_L \frac{V}{p_H} + (1 - p_L)0 - B = p_L \frac{V}{p_H} - B$
- where $p_H > p_L$ (higher effort in the first case) and B diverted from the project to other purposes.

The borrowers's project

Expected net present values

- The expected net present values at time $t = 0$, using the market rate of return as a discount factor, depending on the borrowers actions, are

$$NPV_H = -I + \frac{V}{R},$$

$$NPV_L = -I + \frac{p_L \frac{V}{p_H} - B}{R},$$

- We have $NPV_L < 0 < NPV_H$ if and only if

$$p_L \frac{V}{p_H} - B < RI < V$$

  meaning that project L is bad and project H is good.

Contracts with moral hazard

- A **contract** specifies a spliting of the returns between the lender and the borrower

$$V = V^l + V^b \tag{SPL}$$

- As is common in principal-agent models, to solve the moral hazard problem we introduce two constraints
  - the **participation constraint**: the lender is only interested in signing the contract if he receives the market rate of return on the loaned funds

$$V^l = R(I - W^b) \tag{PC}$$

  - the **incentive compatibility constraint**: the borrower should have the "skin in the game" (good action should be better than bad action)

$$p_H \frac{V^b}{p_H} = V^b \geq p_L \frac{V^b}{p_H} - B \tag{IC}$$

The friction: borrowing constraint

- Equations (SPL) and (IC) imply a **limited pledgeability constraint** ($V^b = V - V^l \geq \frac{P_L}{P_H}(V - V^l) - B$):

$$V^l \leq \bar{V} \equiv V + \frac{p_H}{p_H - p_L} B \tag{LP}$$

  this is the maximum payoff that the borrower can promise to the lender.

- Next we define : the maximum pledgeable payoff that the borrower can offer

$$\bar{v} \equiv \frac{\bar{V}}{I}$$

  ($\bar{V}$ is exogenous).

- The return of the investment to the lender is

$$R\,\theta^l = -R\theta^b = R(I - W^b)$$

The friction: borrowing constraint

- Implication 1 : considering equations (PC) and (LP) then $R(I - W^b) \leq \bar{v}I$ or
$$\theta^b = I - W^b \leq \frac{\bar{v}I}{R} \tag{BC}$$
that is: there is a **borrowing constraint**
- Implication 2: equivalently there is a **collateral requirement**:
$$W^b \geq \bar{W} \equiv I\left(1 - \frac{\bar{v}}{R}\right) \tag{CR}$$
the lender will only finance the project if the borrower has a minimum wealth. If $W^b < \bar{W}$ there will be no finance.

The borrower problem

The problem

- Question: which contract would be optimal to the borrower ?
- We assume that the borrower utility function is linear and that $\beta^l = 1$ (risk neutrality and no impatience). This is equivalent to assuming that he maximizes the cash flow from the project.
- The **borrower investment problem**: seeks to maximize the cash flow from investment subject to the borrowing constraint (BC)

$$\max_I \left\{ vI - R(I - W^b) : \; R\big(I - W^b\big) \leq \bar{v}I, \; I \geq 0 \right\}$$

we denote the payoff of the investment by $v = V/I$.

The borrower problem
Solution

- The f.o.c (optimality and complementarity slackness) are:

$$v - R + \lambda(\bar{v} - R) + \mu I = 0$$

$$\lambda\left(\bar{v}I - R(I - W^b)\right) = 0,\ \lambda \geq 0,\ I \leq \frac{R}{R - \bar{v}} W^b$$

$$\mu I = 0,\ \mu \geq 0,\ I \geq 0$$

- A solution exits if and only if

$$\bar{v} < R < v$$

meaning that there is need to financing $\bar{v} < R$ and the project is worthwhile ($v > R$)

- The optimal investment is

$$\boxed{I^* = \frac{R}{R - \bar{v}} W^b > 0}$$

Equilibrium rate of return

Market equilibrium

- From the lender's problem we derived the **supply of liquidity**

$$\theta^l = \frac{\beta}{1+\beta} W^l$$

- From the borrower's problem we have the **demand for liquidity**

$$-\theta^b = I^* - W^b = \frac{\bar{v}}{R - \bar{v}} W^b > 0$$

- Market equilibrium condition

$$\theta^l + \theta^b = 0$$

Equilibrium interest rate with moral hazard

- Then the equilibrium interest rate r^* is

$$\boxed{R^* = 1 + r^* = \bar{v}\left(1 + \left(\frac{1+\beta}{\beta}\right)\frac{W^b}{W^l}\right)}$$

  - increases with W^b: more wealth from the borrower means more investment and more financing from the lender
  - decreases with W^l: higher liquidity in the economy increases the supply of funds.

- In a **frictionless** economy the equilibrium interest rate would be

$$R = \frac{1}{\beta}$$

Equilibrium interest rate with moral hazard

- Interpretation: in a economy **with informational financial frictions** there is a balance sheet effect on the interest rates: they can be low if there is excess liquidity from the lenders and low net worth (v.g., because of excess leverage) from the borrowers.
- Defining **leverage** by the ratio between borrowing to assets then

$$\ell = -\frac{\theta^b}{W^b} = \frac{\bar{v}}{R - \bar{v}}$$

  we see there is a negative relationship between the equilibrium R and ℓ

- Leverage decreases (increases) with net worth of borrowers W^b (lenders W^l)

References

(Holmström and Tirole, 2011, chap 1)

Holmström, B. and Tirole, J. (2011). *Inside and Outside Liquidity*. MIT Press.