Foundations of Financial Economics *2020/21*
Problem set 7

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16.4.2021

1 Production

1. Consider a two-period production economy where $Y_t = A_t K_t$, where K_t is the capital stock at the beginning of period $t = 0, 1$. Productivity is deterministic at $t = 0$ but it is stochastic at $t = 1$ such that $A_0 = a_0 > 0$, and $A_1 = (a_{1,1}, \ldots, a_{1,N})$. Assume there is an Arrow-Debreu economy with homogeneous agents in which the representative agent has a intertemporally additive discounted von-Neumann-Morgenstern utility functional with $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ for $\theta > 0$.

   (a) Specify the representative agent's problem. Find the first order conditions for an optimum. Justify.

   (b) Define the general equilibrium for this economy and find the stochastic discount factor. Provide an intuition for your result.

2. Consider a two-period production economy where the supply of the good at $t = 0$ is $y_0 = K_0$ and in period 1 is $Y_1 = (y_{1,s})_{s=1}^{N}$, where $y_{1,s} = (1 + \gamma_s)K_1$ with K_t denoting the capital stock at the beginning of period $t = 0, 1$. Assume there is an Arrow-Debreu economy with homogeneous agents in which the representative agent has a intertemporally additive discounted von-Neumann-Morgenstern utility functional with a logarithmic Bernoulli utility function.

   (a) Specify the representative agent's problem. Find the first order conditions for an optimum. Justify.

   (b) Define the general equilibrium for this economy and find the stochastic discount factor. Provide an intuition for your result, by comparing with an endowment economy.

   Solution:

   a) The agent's problem

$$\max_{c_0,(c_{1,s})} \ln c_0 + \beta \sum_{s=1}^{N} \pi_s \ln c_{1,s} \ : \ c_0 + \sum_{s=1}^{N} q_s c_{1,s} = y_0 + \sum_{s=1}^{N} q_s (1+\gamma_s)(2K_0 - c_0)$$

The f.o.c.:

$$\frac{1}{c_0} = \frac{\beta \pi_s}{c_{1,s} q_s} \left(\sum_{s=1}^{N} q_s (1+\gamma_s) + 1) \right)$$
$$c_0 + \sum_{s=1}^{N} q_s c_{1,s} = y_0 + \sum_{s=1}^{N} q_s (1+\gamma_s)(2K_0 - c_0)$$

b) Equilibrium stochastic discount factor $m_s = \dfrac{\beta}{1+\gamma_s}\left(\dfrac{1}{1-\beta}\right)$. This is larger by a factor $1 + \dfrac{1}{\rho}$ as regards the endowment economy.