Foundations of Financial Economics *2020/21*
Problem set 8

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1 Heterogeneous agents

1. Consider an Arrow-Debreu endowment economy in which the information tree is binomial with two periods and two states of nature at time $t = 1$. There are two consumers (indexed by $i = 1, 2$) which have both intertemporally additive preferences, with symmetric Bernoulli functions, $u(c^i) = \frac{(c^i)^{1-\theta}}{1-\theta}$ for $\theta > 1$), and probabilities associated to the two states of nature, but are heterogenous as regards discount factors $\beta^i = \frac{1}{1+\rho^i}$, where $\rho^i > 0$ and endowments $\{y^i_t\}_{t=0}^1$.
   (a) Define Arrow-Debreu general equilibrium for this economy. Find it explicitly. Caracterize it as regards existence and uniqueness.
   (b) Assume instead that agents are also symmetric as regards their intertemporal discount factors. Find the Arrow-Debreu equilibrium in this case.
   (c) Discuss the results you have obtained in the former two questions. Focus on their dependency as regards the symmetry of impatience and the assumptions as regards the endowments (at the individual and the aggregate levels).

2. Consider an Arrow-Debreu economy with two periods and two states of nature for the last period. There are two agents in the economy who are heterogeneous as regards the subjective probabilities associated to the two states of nature and are homogeneous as regards preferences and endowments. The problem for agent $i = 1, 2$ is to maximise the intertemporal utility functional

$$E^i\left[\sum_{t=0}^{1}\beta^t \ln\left(c^i_t\right)\right]$$

   subject to the constraint $c^i_0 + q_1 c^i_{11} + q_2 c^i_{12} = y_0 + q_1 y_{11} + q_2 y_{12}$, where q_s are the AD prices, c^i_{ts} and y_{ts} denote the consumption and the endowment for agent i at t for the state of nature s, respectively.
   (a) Define the AD general equilibrium for this economy.

(b) Solve the agent problem.

(c) Find the AD equilibrium prices and stochastic discount factors. Provide an intuition for your results (hint: compare with an analogous model in which there is homogeneity in information).

(d) What would be the consequences of introducing idiosyncratic uncertainty in the endowments of both agents ?

3. Consider an Arrow-Debreu (AD) economy with an information tree with two periods and $N > 1$ states of nature for the last period. There are $I > 1$ agents in the economy who are heterogeneous as regards the subjective probabilities associated to the two states of nature, π_s^i, and are homogeneous as regards preferences and endowments. The problem for agent $i \in \{1, \ldots, I\}$ is to choose the optimal consumption sequence $\{c_0^i, C_1^i\}$, with $C_1^i = (c_{1,1}^i, \ldots, c_{1,N}^i)$, to maximise the intertemporal utility functional

$$U^i(c_0^i, C_1^i) = \ln(c_0^i) \ + \beta \sum_{s=1}^{N} \pi_s^i \ln(c_{1,s}^i)$$

Â subject to the constraint $c_0^i + \sum_{s=1}^N q_s c_{1,s}^i = y_0 + \sum_{s=1}^N q_s y_{1,s}$, where q_s are the AD prices, and $y_{t,s}$ denotes the endowment for agent i at time t for the state of nature s.

(a) Define the AD general equilibrium for this economy.

(b) Solve agent i's problem.

(c) Find the equilibrium AD prices. Provide an intuition for your results (hint: compare with an analogous model in which there is homogeneity in information).

(d) Discuss the consequences of introducing heterogeneity in the endowments of both agents. Focus on both micro and aggregate consequences.

Solutions:

b) The Lagrangean for agent i is

$$\mathcal{L}^i = \ln(c_0^i) + \beta \sum_{s=1}^{N} \pi_s^i \ln(c_{1,s}^i) + \lambda^i \left(h - c_0^i - \sum_{s=1}^{N} q_s c_{1,s}^i \right)$$

Â where $h = y_0 + \sum_{s=1}^N q_s y_{1,s}$ is wealth of agent i, which is equal for all agents. The solution for agent i problem is

$$c_0^i = \frac{h}{1+\beta}$$
$$c_{1,s}^i = \frac{\beta \pi_s^i}{q_s} \frac{h}{1+\beta}, \ s = 1, \ldots, N$$

c) From the market equilibrium conditions

$$\sum_{i=1}^{I} c_0^i = \sum_{i=1}^{I} y_0 = I y_0$$
$$\sum_{i=1}^{I} c_{1,s}^i = \sum_{i=1}^{I} y_{1,s} = I y_{1,s}, \; s = 1, \dots, N$$

Â we find

$$\frac{h}{1+\beta} = y_0 \; \frac{\beta}{q_s} \frac{h}{1+\beta} \sum_{i=1}^{I} \pi_s^i = I y_{1,s}$$

Â Defining the average probability of state s among the I agents by

$$\bar{\pi}_s = \frac{\sum_{i=1}^{I} \pi_s^i}{I} = \frac{\pi_s^1 + \dots \pi_s^i + \dots + \pi_s^I}{I}$$

Â we find the equilibrium state price

$$q_s = \beta \frac{y_0}{y_{1,s}} \bar{\pi}_s$$

Â This is formally similar to the case in which there is homogeneous information $\pi_s^i = \pi_s$ for all $i \in \{1, \dots, I\}$ where the probability of state s is substituted by the (population) average probability of state s.