Universidade de Lisboa
Instituto Superior de Economia e Gestão
Departamento de Economia

Mestrado em Economia Monetária e Financeira (Master in Monetary and Financial Ecoinomics)
Fundamentos de Economia Financeira (Foundations of Financial Economics)
2020-2021

Instructor: Paulo Brito
Exame: **Época normal** (First exam)
31.5.2021 (18.00h-21.00h, Biblioteca)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. `[6 points (2,2,2)]` Consider a deterministic, two-period, representative-agent finance economy where the initial financial wealth is zero, the flow of endowment is $\{y_0, y_1\}$ and the intertemporal utility function is
$$U(c_0, c_1) = \frac{c_0^{1-\theta} - 1}{1-\theta} + \beta \frac{c_1^{1-\theta} - 1}{1-\theta},\ 0 < \beta < 1,\ \theta > 0$$
   a) Characterize the implicit behavioral assumptions.
   b) Specify the agent's problem. Solve the representative agent problem.
   c) Define the general equilibrium. Find the equilibrium asset return. Provide an intuition.

2. `[6 points (2,2,2)]` For a two period binomial-tree with two states of nature, let a financial market be characterized by the following price vector and $(N \times K)$ payoff matrix
$$\mathbf{S} = \left(1, \frac{1}{R}\right), \text{ and } \mathbf{V} = \begin{pmatrix} R+\epsilon & 1 \\ R-\epsilon & 1 \end{pmatrix},$$
   where $R > 1$ and ϵ can take any real value.
   (a) Under which conditions we may have arbitrage opportunities ? Justify.
   (b) From now on assume there are no arbitrage opportunities. Find the state prices.
   (c) Consider a worker facing a prospect of unemployment at period $t = 1$ and expecting to earn a contingent wage $Y^{un} = \begin{pmatrix} \phi \\ 0 \end{pmatrix}$ for $\phi > 0$. Assume there is an institution which can insure its income such that his wage can become state independent, that is $Y^{in} = \begin{pmatrix} \phi \\ \phi \end{pmatrix}$. This institution hedges the difference $Y^{in} - Y^{un}$ by building a replicating portfolio. Find the replicating portfolio and the cost of providing insurance. Discuss your result.

3. `[8 points (3,3,2)]` Consider an homogeneous agent endowment finance economy in which there is a risk-free asset, with a return equal to $R^f = 1 + r$, and a risky asset with return $R = (1 + \varrho, 1 - \varrho)^\top$, for $\varrho > 0$. The endowment process is $Y = \{y_0, Y_1\}$ where $Y_1 = ((1+\gamma)y_0, (1-\gamma)y_0)^\top$ for $0 < \gamma < 1$. The representative consumer has the intertemporal utility functional
$$U(c_0, C_1) = \log(c_0) + \beta\, \mathbb{E}\big[\log(C_1)\big], \text{ where } 0 < \beta < 1.$$
   (a) Characterize the behavior of the agent which is implicit in the utility functional.
   (b) Define the dynamic stochastic general equilibrium for this economy. Find the equilibrium stochastic discount factor.
   (c) Find the equilibrium rates of return for the risk free and the risky asset, i.e., r and ϱ. Discuss why it is possible to determine them uniquely.