

Instructor: Paulo Brito

Exam: **Re-sit Exam**

1.7.2021 (15.00h-18.00h, room 101 F1)

Closed book exam. No auxiliary material (on paper, electronic or any other form) is allowed.

1. [6 points (1,2,2,1)] Consider a deterministic, two-period, representative-agent finance economy where the initial asset stock is zero, the flow of endowment is $\{y_0, y_1\}$, where $y_1 = (1 + \gamma)y_0$, with $\gamma \geq 0$. The intertemporal utility functional is

$$U(c_0, c_1) = \log c_0 + \beta \log \left(\left(\frac{c_1}{c_0} \right)^\zeta \right), \quad 0 < \beta < 1$$

- (a) Characterize the implicit behavioral assumptions in the utility functional.
 - (b) Specify and solve the representative agent problem. Characterize and provide an intuition to the savings behavior of the household.
 - (c) Define the general equilibrium. Find the equilibrium asset return. Characterize and provide an intuition for its properties.
 - (d) Which type of interest rate theory justifies this model ? Does this explain the historical negative correlation between the risk free interest rate and the rate of economic growth ? Provide an intuition for what is missing in this model to account for that negative correlation.
2. [6 points (2,2,2)] Assume there is a financial market with two assets, one risky asset and one riskless asset with prices and payoffs

$$\mathbf{S} = \left(\frac{1}{1+i} \quad s \right), \quad \mathbf{V} = \begin{pmatrix} 1 & d_h \\ 1 & d_l \end{pmatrix},$$

where $i > 0$ and $d_h > d_l$ are the payoffs for the risky asset in the two states of nature h, l .

- (a) Find the conditions under which there are no arbitrage opportunities and the market is complete. (From now on assume the condition you have just found.)
 - (b) Introduce a European put option with exercise price d_0 , satisfying $d_l < d_0 < d_h$. By constructing a replicating portfolio, find the option's price under the assumption of absence of arbitrage opportunities.
 - (c) Assume that the two states of nature have equal probabilities. In this model the Sharpe index is equal to the Hansen-Jaganathan bound. Check it and provide an intuition why this is the case
3. [8 points (1,2,3,2)] Consider an Arrow-Debreu endowment economy in which the information tree is binomial with two periods and two states of nature at time $t = 1$. There are two consumers (indexed by $i = 1, 2$) with homogeneous preferences represented by the utility functional

$$U(c_0^i, C_1^i) = \ln(c_0^i) + \beta \sum_{s=1}^2 \pi_s \ln(c_{1s}^i).$$

However agents are heterogeneous as regards their endowments: agent $i = 1$ receives $\{y_0^1, Y_1^1\} = \{\frac{\bar{y}}{2}, (\bar{y}, 0)\}$ and agent $i = 2$ receives $\{y_0^2, Y_1^2\} = \{\frac{\bar{y}}{2}, (0, \bar{y})\}$, for $\bar{y} > 0$.

- (a) Characterize the economy as regards the type of uncertainty.

- (b) Characterize the behavioral assumptions which are implicit in the utility functional.
- (c) Define and find the Arrow-Debreu equilibrium of this economy (hint: find explicitly the consumption processes for agents $i = 1$ and $i = 2$, write the market equilibrium conditions, and, from them, find the stochastic discount factor).
- (d) Is there complete insurance, at the equilibrium, in this economy ? Does this mean that the consumption allocations across time will be equal for the two consumers ? Provide an intuition for your results.