# Foundations of Financial Economics
# Financial frictions: moral hazard

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May 13, 2022

# This lecture

- General equilibrium with moral hazard: the Holmstrom Tirole model
- We consider again the ”internal” finance model: demand and supply of funds between heterogeneous agents
- Main difference from the benchmark model: **asymmetric information**
- In this case, we consider **moral hazard** (or the principal-agent model): one party does not observe the **actions** of the other
- This generates a **financial friction**: a borrowing constraint
- And a balance effect: **the distribution of wealth between agents has an effect on the interest rate**
- This provides a solid theoretical underpinning to a old theory of interest rates: the loanable fund theory.

# Topics

- The lender's problem
- Contracts in the presence of moral hazard
- Financial friction: borrowing constraint
- The borrower's problem
- Equilibrium interest rate.
- **Simplifying assumption**: the resources of the economy take the form of financial wealth distributed at the beginning of period 0.

# The lender

# The lender's problem

## Assumptions

- Has **liquid net worth** $W^l$, that is higher than the desired consumption at time $t = 0$, and it is the only source of finance for consumption at time $t = 1$.
- **Lends $\theta^l$ through a debt contract** in which the return at time $t = 1$ is **risk-free**. Therefore consumption at time $t = 1$ is risk free.
- The **lender's problem** is

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \theta^l = W^l, \ c_1^l = R\theta^l$$

  where $R$ is the return on the asset.
- The Bernoulli utility function is concave: $u''(c) < 0 < u'(c)$

# The lender's problem

## Solution

- Equivalently

$$\max_{c_0^l, c_1^l} u(c_0^l) + \beta u(c_1^l) \text{ s.t } c_0^l + \frac{c_1^l}{R} = W^l$$

- Assuming a log utility function the solution is

$$c_0^l = \frac{1}{1+\beta} W^l, \; c_1^l = \frac{\beta R}{1+\beta} W^l$$

- The demand for the asset, or the **liquidity supply**, is

$$\boxed{\theta^l = \frac{c_1^l}{R} = \frac{\beta}{1+\beta} W^l}$$

# The borrower

# The borrowers' project

- Investment $I$ in a project.
- The net payoff of the project depends from the borrowers' actions (which are random from the perspective of the lender);
  - The borrower can follow one of the two courses of action (**not observable by the lender**):
    - put high effort and use all the resources in the project
    - put low effort and divert resources from the project (or having a more inefficient management)
  - The probability of success depends on the effort level ($\pi_H > \pi_L$).

# The borrowers's project

## Expected returns

- The expected returns, obtained at period $t = 1$ from the courses of action are: with expected returns
  - Good project: $E[V_H] = \pi_H \frac{V}{\pi_H} + (1 - \pi_H)0 = V$
  - Bad project: $E[V_L] = \pi_L \frac{V}{\pi_H} + (1 - \pi_L)0 - B = \pi_L \frac{V}{\pi_H} - B$
- where $\pi_H > \pi_L$ (higher effort in the first case) and $B$ diverted from the project to other purposes.

# The borrowers's project

## Expected net present values

- The expected net present values at time $t = 0$, using the market rate of return as a discount factor, depending on the borrowers actions, are

$$NPV_H = -I + \frac{E[V_H]}{R} = -I + \frac{V}{R},$$

$$NPV_L = -I + \frac{E[V_L]}{R} = -I + \frac{\pi_L \frac{V}{\pi_H} - B}{R}$$

- We have $NPV_L < 0 < NPV_H$ if and only if

$$\pi_L \frac{V}{\pi_H} - B < R\, I < V$$

meaning that project $L$ is bad and project $H$ is good.

# Financing the project

- The borrower has **net worth** $W^b$
- If $I \geq W^b$ the borrower needs financing from the lender

$$\theta^b = W^b - I < 0$$

- In order to get financing borrower and the lender **need to sign a contract**

# Contracts with moral hazard

- A **contract** specifies a spliting of the returns between the lender and the borrower

$$V = V^l + V^b \tag{SPL}$$

- As is common in principal-agent models, to solve the moral hazard problem we introduce two constraints
  - the **participation constraint**: the lender is only interested in signing the contract if he receives the market rate of return on the loaned funds

$$V^l = R(I - W^b) \tag{PC}$$

  - the **incentive compatibility constraint**: the borrower should have "skin in the game" (good action should be better than bad action) if

$$\pi_H \frac{V^b}{\pi_H} = V^b \geq \pi_L \frac{V^b}{\pi_H} - B \tag{IC}$$

# The friction: borrowing constraint

- Equations (SPL) and (IC) imply a **limited pledgeability constraint**:

  $$V^l \leq \bar{V} \equiv V + \frac{\pi_H}{\pi_H - \pi_L} B \qquad \text{(LP)}$$

  (because $V^b = V - V^l \geq \frac{\pi_L}{\pi_H}(V - V^l) - B$)

  This is the maximum payoff that the borrower can promise to the lender.

- Next we define : the maximum pledgeable return that the borrower can offer

  $$\bar{v} \equiv \frac{\bar{V}}{I}$$

  ($\bar{V}$ is exogenous).

# The friction: borrowing constraint

- The from of income from the investment to the lender is

$$R\,\theta^l = -R\theta^b = R(I - W^b)$$

- From the previous constraints, (PC) and (LP), we have two equivalent requirements
  1. as then $R(I - W^b) \leq \bar{v}I$ then

$$\theta^b = I - W^b \leq \frac{\bar{v}I}{R} \tag{BC}$$

     that is: there is a **borrowing constraint**
  2. equivalently there is a **collateral requirement**:

$$W^b \geq \bar{W} \equiv I\left(1 - \frac{\bar{v}}{R}\right) \tag{CR}$$

     the lender will only finance the project if the borrower has a minimum wealth. If $W^b < \bar{W}$ there will be no finance.

# The borrower problem

## The problem

- Question: which contract would be optimal to the borrower ?
- **Assumption**: the borrower utility function is linear and that $\beta^l = 1$ (risk neutrality and no impatience).
  (this is equivalent to assuming that he maximizes the cash flow from the project.)
- The **borrower investment problem**: seeks to maximize the cash flow from investment subject to the borrowing constraint (BC)

$$\max_{I} \left\{ vI - R(I - W^b) : \; R\big(I - W^b\big) \leq \bar{v}I, \; I \geq 0 \right\}$$

we denote the payoff of the investment by $v = V/I$.

# The borrower problem

## Solution

- The f.o.c (optimality and complementarity slackness) are:

$$v - R + \lambda(\bar{v} - R) + \mu I = 0$$

$$\lambda\left(\bar{v}I - R(I - W^b)\right) = 0,\ \lambda \geq 0,\ I \leq \frac{R}{R - \bar{v}} W^b$$

$$\mu I = 0,\ \mu \geq 0,\ I \geq 0$$

- A solution exits if and only if

$$\bar{v} < R < v$$

meaning that there is need to financing $\bar{v} < R$ and the project is worthwhile ($v > R$)

- The optimal investment is

$$\boxed{I^* = \frac{R}{R - \bar{v}} W^b > 0}$$

# Equilibrium rate of return

# Market equilibrium

- From the lender's problem we derived the **supply of liquidity**

$$\theta^l = \frac{\beta}{1+\beta} W^l$$

- From the borrower's problem we have the **demand for liquidity**

$$-\theta^b = I^* - W^b = \frac{\bar{v}}{R-\bar{v}} W^b > 0$$

- **Market equilibrium condition**

$$\theta^l + \theta^b = 0$$

# Equilibrium interest rate with moral hazard

- Then the **equilibrium return** is

$$\boxed{R^{eq} = R^{eq}(\underset{+}{W^b}, \underset{-}{W^l}) = \bar{v}\left(1 + \left(\frac{1+\beta}{\beta}\right)\frac{W^b}{W^l}\right)}$$

  - increases with $W^b$: wealthier borrowers increase supply of funds (which increases investment)
  - decreases with $W^l$: higher liquidity in the economy decreases the demand for funds

- In a **frictionless** economy the equilibrium interest rate would be

$$R = \frac{1}{\beta}$$

(Obs.: this is the case in which there is no aggregate uncertainty, because $V$ is deterministic)

# Equilibrium interest rate with moral hazard

- Interpretation: in a economy **with informational financial frictions** there is a balance sheet effect on the interest rates: they can be low if there is excess liquidity from the lenders and low net worth (v.g., because of excess leverage) from the borrowers.
- The distribution of wealth has an effect on the return of assets

# Equilibrium leverage

- **Leverage** is measured by the ratio between borrowing to assets

$$\ell = -\frac{\theta^b}{W^b} = \frac{\bar{v}}{R - \bar{v}}$$

- Then equilibrium leverage also depends on the distribution of wealth

$$\ell^{eq} = \ell^{eq}\big(\underset{-}{W^b}, \underset{+}{W^l}\big)$$

  Equilibrium leverage:

  - decreases with net worth of borrowers $W^b$ (more own financing by borrowers)
  - increases with net worth of lenders $W^l$ (external financing cheaper)

# References

(Holmström and Tirole, 2011, chap 1)

Holmström, B. and Tirole, J. (2011). *Inside and Outside Liquidity*. MIT Press.