

Universidade de Lisboa  
 Instituto Superior de Economia e Gestão  
 Ph.D. in Economics  
 Advanced Macroeconomics

**Part I**

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Year: 2021-2022

Exam: **Second Exam**

Date: 2.2.2022

Schedule: 18:00-21:00

**Instructions:**

- This is an open book exam.
- The use of electronic devices is forbidden.
- The answers to each part of the exam should be provided in a separate set of pages.
- Answers should be concise. Any answer not strictly related to the question may receive a negative evaluation.

• The two questions of Part I have multiple subquestions and they require some cleverness, so be careful. The subquestions build on each other, so it is important to take your time and get a subquestion right before moving on to the next subquestion.

**Part I**

[7 pts] **1.** Consider a version of our model with firms and households. Households own the firms and each period receive profits  $\Pi_t$ . Households pay consumption taxes when they purchase goods. So, instead of paying the price per unit,  $P_t$ , they pay  $(1 + \tau_c)P_t$ . The purchases of private consumption,  $C_t$ , have to be made with cash, i.e.  $(1 + \tau_c)P_t C_t = vM_t^h$ , where  $P_t$  is the price level,  $v$  is a parameter representing velocity, and  $M_t^h$  the money holdings of the households. Households own the factors of production, which they rent to firms. Labor income,  $w_t L_t$ , is not taxed, but capital income,  $r_t K_t$ , is taxed at rate  $\tau_k$ . The tax receipts are rebated lump-sum to households. In this case the representative household's problem becomes

$$\max_{\{C_t, L_t, M_{t+1}^h, B_{t+1}, K_{t+1}\}_{t=1,2}} u(C_1) - v(L_1) + \beta [u(C_2) - v(L_2)] + \beta^2 V(M_3^h, B_3, K_3) \text{ subject to}$$

$$vM_t^h \geq (1 + \tau_c)P_t C_t, \text{ for } t = 1, 2 \text{ and}$$

$$\Pi_t + P_t \{w_t L_t + (1 - \tau_k)r_t K_t - \delta K_t\} + [M_t^h - (1 + \tau_c)P_t C_t] + B_t + T_t \geq M_{t+1}^h + q_t B_{t+1} + P_t [K_{t+1} - K_t] \text{ for all } t \leq 2.$$

Firms hire labor  $\bar{L}_t$  and capital  $\bar{K}_t$  to maximize their profits

$$\Pi_t = P_t [Z_t \bar{L}_t]^{1-\alpha} \bar{K}_t^\alpha - w_t \bar{L}_t - r_t \bar{K}_t.$$

Additionally, firms must use cash,  $M_t^f$ , to pay wages

$$M_t^f \geq w_t P_t \bar{L}_t.$$

- A) [1 pt.] Define formally the competitive equilibrium of this economy.
- B) [1 pt.] Write down the Lagrangian for the representative household and determine the first-order conditions for the optimal levels of consumption, labor, money, bonds and capital.
- C) [1 pt.] Derive the first-order conditions of the representative firm's problem.
- D) [2 pt.] Assume that we have log preferences over consumption, so  $u(C_t) = \log C_t$ , and a power function for labor,  $v(L_t) = \frac{L_t^{1+\gamma}}{1+\gamma}$ . Finally, assume that the cash-in-advance constraint binds, and that money and productivity are constant. Given these assumptions, compute the stationary competitive equilibrium.

E. [2 pt.] The government is contemplating a fiscal policy change. A capital tax reduction accompanied by a change in the consumption tax so that the government revenue is unchanged. What are the effects of this change in the fiscal policy on welfare. Use the equilibrium equations to explain.

[3 pts] **2.** Imagine a two period economy with two types of households:  $A$  and  $B$ . There are  $N_A$  households of type  $A$  and  $N_B$  households of type  $B$ . Both have the same preferences,  $U_i = \sum_{t=1}^2 \beta^t u(C_{i,t})$ , for  $i = A, B$ , with  $0 < \beta < 1$ ,  $u' > 0$  and  $u'' < 0$ . The households differ in their endowment of a perishable good,  $Y_{i,t}$ , for  $i = A, B$ , and  $t = 1, 2$ . Type  $A$  households have a lot of endowment in the first period, and very little in the second, i.e.  $Y_{A,1} \gg Y_{A,2}$ . Conversely, type  $B$  households have very little endowment in the first period but a lot in the second, i.e.  $Y_{B,1} \ll Y_{B,2}$ .

A) [2 pt.] Write down the competitive equilibrium equations that determine the consumptions and the real interest rate:  $(C_{A,1}, C_{B,1}, C_{A,2}, C_{B,2}, r)$ .

B) [1 pt.] What would you expect to happen to the interest rate if the government forbids agents of type  $B$  to access the credit market? Explain using the first order conditions of the consumers problem.