Universidade de Lisboa
Instituto Superior de Economia e Gestão

PhD in Economics
**Advanced Macroconomics**
**Part II**

Lecturers: Paulo Brito
Year: **2021-2022**
Exam: **First Exam**
Date: 17.1.2022
Schedule: 18:00-21:00

**Instructions**:

- This is an open book exam. The use of any electronic devise is forbidden.
- Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Please write your answer for every part in an independent set of pages.

## Part II

**1** [5 points: 1, 2, 1, 1] Consider an uncertain lifetime finite horizon utility functional in which the terminal time has a Poisson distribution with instantaneous mortality rate $\mu > 0$. We can prove that the utility functional is

$$\mathsf{U}[c] = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} \; e^{-(\rho+\mu)\,t}\, dt$$

where $\rho > 0$ and $1/\theta > 0$, are the rate of time preference and the elasticity of intertemporal substitution, and $c(t)$ is the consumption flow at time $t$. The household can apply its savings to investment into a bond, whose market rate of return is $r > 0$. Additionally, the household can insure its lifetime income by having access to a Yaari annuity market which pays a rate of return $\mu$ against its remaining stock of financial wealth in the case of death. Furthermore, the household has a constant flow of non-financial income $y$. With all those assumptions, the budget constraint of the household is

$$\dot{a} = (r+\mu)\, a(t) + y - c(t),$$

where $a(t)$ denotes the financial net-wealth at time $t \geq 0$. It is also assumed that the initial net-wealth level is known and it is positive, $a(0) = a_0 > 0$, and a non-Ponzi game condition should be verified.

(a) Explain why the utility functional takes the previous form. Characterize the intertemporal preferences (hint consider a stationary consumption flow). Provide an intuition.

(b) Solve the household problem, introducing assumptions for the existence of a solution if needed.

(c) Assume that $\rho + \mu = r$. Draw the phase diagram, justifying it characteristics. Discuss its economic intuition.

(d) Assume there is an unanticipated increase in non-financial income, $y$, starting from a steady state. Study the comparative dynamics for this shock, both analytically and geometrically. Discuss your results.

**2**[`5 points: 1, 1, 1, 1, 0.5, 0.5`] Consider a new-Keynesian model with monopolistic competition. The final product market is competitive and the technology of production uses a continuum of intermediate good varieties $j \in [0,1]$. Each variety is produced by a monopolist and uses capital as the only input. In the next questions, items (a) to (c) are dedicated to building up the details for the definition of the general equilibrium and items (d) to (f) and concerned with its representation and characterization.

(a) Each representative household solves the following problem

$$\max_{c(\cdot)} \int_0^\infty \ln c(t)\, e^{-\rho t}\ dt, \text{ subject to } \dot{w} = r(t)\, w(t) - c(t)$$

for $\rho > 0$, $w(0) = w_0$ fixed, and $\lim_{t\to\infty} e^{-R(t)}\ w(t) \geq 0$, with $R(t) = \int_0^t r(s)\, ds$. The variables $c(t)$, $w(t)$, and $r(t)$ denote consumption, net asset position, and the asset market rate of return, respectively. Find the optimality conditions.

(b) The problem of the producer of the final good is

$$\max_{\mathbf{x}(t)} \left\{ y(t) - \int_0^1 p(j,t)\, x(j,t)\, dj, \text{ subject to } F(\mathbf{x}(t)) = y(t) \right\}$$

where $F(\mathbf{x}(t))$, and $y(t)$ denote the production function, and the output at time $t$. The production function $F(\mathbf{x}(t)) = \left( \int_0^1 A\, x(j,t)^{1-\mu}\ dj \right)^{\frac{1}{1-\mu}}$, where $A$ is a productivity parameter, and $0 < \mu < 1$. Prove that the solution is $x^*\ (j,t) = \left(\frac{p(j,t)}{A}\right)^{-\frac{1}{\mu}} y(t)$.

(c) Assuming that the production function for the monopolist producing variety $j$ is $x(j,t) = k(j,t)$, where $k(j,t)$ is the stock of capital, and that the cost per unit of investment is $\xi$, the $j$-monopolist problem is

$$\max_{k(j,\cdot)} \int_0^\infty \left(\pi(j,t) - \xi\, \dot{k}(j,t)\right) e^{-R(t)} \; dt \text{ given } \; k(j,0) = k(0)$$

where $R(t) = \int_0^t r(s)\, ds$ and $r(t)$ are the discount rate and the asset market rate of return at time $t$. Show that the profit is $\pi(j,t) = A\, k(j,t)^{1-\mu} \; y(t)^\mu$. Prove that the optimality condition for the $j$-monopolist is

$$(1-\mu)\, Ak(j,t)^{-\mu} \; y(t)^\mu \; = \xi\, r(t) \text{ for each } \; t \in [0,\infty).$$

(d) Define the DGE (dynamic general equilibrium) for this economy. Show that it can be characterized by the problem,

$$\begin{aligned}
\dot{k} \; &= \bar{r} \;\; k - c, \text{ where } \; \bar{r} \;\; = \frac{1-\mu}{\xi} A^{\frac{1}{1-\mu}} \\
\dot{c} \; &= c\,(\bar{r} - \rho) \\
k(0) &= k_0 \text{ fixed} \\
\lim_{t\to\infty} \frac{k(t)}{c(t)} \; e^{-\rho\, t} \; &= 0
\end{aligned}$$

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(e) Characterize the equilibrium dynamics for $\bar{r} \; = \rho$ and for $\bar{r} \; > \rho$.

(f) Starting from $\bar{r} \;\; = \rho$, determine the dynamic effects from a permanent and non-anticipated increase in the productivity, $A$.