Universidade de Lisboa
Instituto Superior de Economia e Gestão

PhD in Economics
Advanced Macroconomics
Part II

Lecturers: Paulo Brito
Year: **2021-2022**
Exam: **Re-sit Exam**
Date: 2.2.2022
Schedule: 18:00-21:00

Instructions:

- This is an open book exam. The use of any electronic devise is forbidden.
- Your answer should be concise, objective and specific. Any notation, calculation, motivation, discussion or explanation **not strictly related to the specific question** it tries to address will either not be considered or have a negative assessment.
- Please write your answer for every part in an independent set of pages.

Part II

1 [`6 points:1,3,1,1`] Consider a Ramsey model in which labor is endogenous. The constraint of the economy is given by equation $\dot{k} \equiv \dfrac{dk(t)}{dt} = k^{\alpha}\,\ell^{1-\alpha} - c$, where $0 < \alpha < 1$. The central planner optimal allocation is a solution to the HJB equation

$$\rho\, v(k) = \max_{c,\ell} \Big\{ \ln(c) - \psi\,\ell + v'(k)\left(k^{\alpha}\,\ell^{1-\alpha} - c\right) \Big\}.$$

where $\psi > 0$, and $\rho > 0$, with the usual interpretation: k, ℓ and c denote the per-capital capital stock, labor effort, and consumption, respectively, and $v(\cdot)$ is the value function.

(a) Find the optimal policy functions and write the optimal HJB equation.

(b) Using the envelope theorem, and the policy functions you obtained in (a), we can obtain a representation of the optimality conditions as a dynamic system in (k, c),

$$\dot{k} = \frac{R(c)}{\alpha}\, k - c$$
$$\dot{c} = c\left(R(c) - \rho\right),$$

where $R(c) \equiv \alpha \left(\frac{1-\alpha}{\psi\, c}\right)^{\frac{1-\alpha}{\alpha}}$. Prove this.

(c) Draw the phase diagram, assuming that k is a pre-determined variable, and the optimal path is conditionally stable. Explain your reasoning.

(d) Study the effects of non-anticipated, permanent, and constant increases in ψ and in ρ. Provide economic intuitions for your results.

2[`4 points:2,1.5,0.5`] Consider a finance market economy in which there is only one risky asset entitling the holder to a dividend following the process $dD(t) = g\,D(t)\,dt + \sigma\,D(t)\,dW(t)$, where $g > 0$, and $\sigma > 0$ are given constants, and $\big(W(t)\big)_{t\geq 0}$ is a Wiener process. Households have an intertemporal utility functional with form

$$\mathbb{E}_0\left[\int_0^\infty \frac{C(t)^{1-\gamma}-1}{1-\gamma}\, e^{-\rho t}\, dt\right], \text{ for } \rho > 0,\ \gamma > 0,$$

where $C(t)$ is consumption at time t. The household balance sheet is, at time t, is $S(t) = N(t)$, where N and S denote net wealth and the market value of the risky asset. Its financial gain process is

$$dG(t) = S(t)\,\Big(\big(\mu_s(t) + z(t)\big)\,dt + \sigma_s(t)\,dW(t)\Big)$$

where μ_s and σ_s are the market return and volatility parameters for the risky asset, and $z(t) = D(t)/S(t)$ is the dividend-price ratio. There is no non-financial income.

(a) The household problem is to maximize the utility functional subject to the budget constraint $dN(t) = dG(t) - C(t)\,dt$, for $t > 0$, given $N(0) = N_0$, by choosing an optimal path for consumption. Find the solution to its problem.

(b) Assuming that the risky asset is in positive net supply, equal to $D(t)$, define the dynamic stochastic general equilibrium. Find the equilibrium values for $z(t)$, $\mu_s(t)$ and $\sigma_s(t)$.

(c) Discuss and interpret your results.