Advanced macroeconomics 2021-2022
Problem set 2: Ramsey and DGE models

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1 Ramsey: general

1 Discuss the effects of assuming decreasing marginal returns to capital and intertemporal substitution in consumption on the dynamics of the Ramsey model.

2* Discuss the effects of assuming anticipated permanent productivity shocks on the optimal path of the economy, according to the Ramsey model[1].

3* Discuss the effects of effects of non-anticipated temporary productivity shocks on the optimal path of the economy, according to the Ramsey model

2 Ramsey

1 Consider a Ramsey model in which there is depreciation of capital, and the production function is Cobb-Douglas $y = f(k) \equiv A\,k^{\alpha}$, for $A > 0$ and $0 < \alpha < 1$. That is the capital accumulation constraint is $\dot{k} = Ak^{\alpha} - c - \delta\,k$, where $\delta > 0$, where all the variables are in per-capita terms. The instantaneous utility function is $u(c) = \log(c)$ and the rate of time preference is $\rho > 0$.

(a) Solve the Ramsey problem by using the PMP.

(b) Draw the phase diagram.

(c) In this model there is a manifold passing through two steady states, one in which both k and c are positive, and another one in which $c = 0$ and $f(k) = \delta k$ for $k > 0$. Prove that there are admissible trajectories connecting those two points. Explain why those trajectories cannot be optimal.

(d) Perform a comparative dynamics exercise for an increase in δ. Provide an intuition for your results.

[1] Questions are marked with asterisks depending on their degree of difficulty.

2 Consider a Ramsey model in which unfunded government expenditures can exist. The economy's resource constraint is $\dot{k} = Ak^{\alpha} - c - g - \delta\, k$, where $g \geq 0$ is a public transfer, $\delta > 0$, $A > 0$, and $0 < \alpha < 1$. The instantaneous utility function is isoelastic $u(c) = \dfrac{c^{1-\theta} - 1}{1-\theta}$, for $\theta > 0$, and the rate of time preference is $\rho > 0$.

(a) Solve the Ramsey problem by using the PMP.

(b) Is it possible to solve explicitly the Ramsey using the DPP ?

(c) Perform a comparative dynamics exercise for an increase in g. Provide one intuition for your results.

3 DGE

1 Consider a DGE economy in which the utility function is $u(c) = \log(c)$, the rate of time preference is $\rho > 0$, there is a constant number of households N, there is no unemployment, and the technology for firms is CES

$$Y = A\left(\alpha\, K^{\eta} + (1-\alpha)\, L^{\eta}\right)^{\frac{1}{\eta}}$$

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant shock in A. Provide an intuition for your results.

2 Consider a DGE economy in which the utility function is $u(c) = \log(c)$ and the production function is Cobb-Douglas in which the government raises an income tax and has a budget balanced fiscal policy. Denoting per capita government expenditure by g and the tax rate is denoted by τ and both are constant through time. The budget balance rule is $g = \tau\,(r(t)a(t) + w(t))$. Assume that households supply labor inelastically and they have the budget constraint $\dot{a} = (1-\tau)\,(r(t)a(t) + w(t)) - c(t) + g(t)$.

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant increase in g. Provide one intuition for your results.

4 Endogenous labor

1 Consider the following Ramsey model with endogenous labor supply

$$\max_{c(\cdot),\ell(\cdot)} \int_0^\infty \Big(\ln\left(c(t)\right) - \psi\, \ell \Big)\, e^{-\rho}\, dt$$

subject to

$$\dot{k} = A\, k^\alpha\, \ell^{1-\alpha}$$

$$k(0) = k_0 > 0 \text{ given}$$

$$\lim_{t\to\infty} k(t) \geq 0$$

(a) Find the first order conditions for optimality.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant increase in TFP A. Provide one intuition for your results.

2* Consider a Ramsey model with endogenous labour with additively separable preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{c^{1-\theta}-1}{1-\theta} - \psi\, \frac{\ell^{1+\zeta}}{1+\zeta},\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

in which the rate of time preference is $\rho > 0$, the production function is

$$f(k,\ell) = A\, k^\alpha\, \ell^{1-\alpha},\ \text{for } A > 0,\ 0 < \alpha < 1,$$

and there is no capital depreciation.

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

3* Consider a Ramsey model with endogenous labour with KPR preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{\Big(c\left(1-\psi\,\ell^\eta\right)^{1-\theta} - 1}{1-\theta},\ \text{for } \theta > 0,\ \psi > 0,\ \eta > 0$$

in which the rate of time preference is $\rho > 0$, the production function is

$$f(k,\ell) = A\, k^\alpha\, \ell^{1-\alpha},\ \text{for } A > 0,\ 0 < \alpha < 1,$$

and there is no capital depreciation.

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

4* Consider a Ramsey model with endogenous labour with GHH preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{1}{1-\theta}\left(\left(c - \psi\,\frac{\ell^{1+\zeta}}{1+\zeta}\right)^{1-\theta} - 1\right),\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

and

$$f(k,\ell) = A\,k^{\alpha}\,\ell^{1-\ell}$$

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.