

Advanced macroeconomics 2021-2022  
Problem set 3: asset pricing and financial frictions

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## 1 General questions

1. The modern macro-financial literature stresses the importance of financial frictions as a source of macroeconomic fluctuations.
  - (a) What is the meaning of financial frictions ? Which type of financial frictions can we consider ?
  - (b) Why do financial frictions have macroeconomic effects ?
  - (c) Wealth distribution tend to have non-neutral macroeconomic effects in the presence of financial frictions. Why ?
2. Consider a finance economy in which there is heterogeneity in the risk aversion between agents. Consider, for instance, two groups with high and low risk aversion.
  - (a) Can a model with limited participation but homogeneous risk aversion provide an intuition on the characteristics of the general equilibrium in the presence of risk aversion heterogeneity but without constraints in the participation in the asset markets ?
  - (b) Which behavior of the rates of return of the risk free and risky assets should we expect in this case ?
3. Consider a finance economy in which there agents are homogenous in terms of their preferences, but there is limited participation. Assume there are collateral constraints limiting the borrowing capacity of participating agents, for instance  $-B^x(t) \leq \kappa S(t)$ . Which consequences this would have on the characterization of the general equilibrium ?

## 2 Problems

1. Consider an Arrow-Debreu economy in which the representative household has a log utility function, there are two assets and the endowment process is  $dY(t) = (\bar{Y} - y(t)) dt + \sigma dW(t)$ , where  $\bar{Y} > 0$  and  $\sigma > 0$ .

- (a) Specify and solve the household problem.
  - (b) Define and find the general equilibrium.
  - (c) Discuss the properties of the stochastic discount factor.
2. Consider a frictionless finance market economy in which the representative household has a log utility function, there are two assets, one risky asset and one risk free asset. The risky asset entitles the holder to receive a dividend that follows the process  $dD(t) = D(t)\sigma dW(t)$ , where  $\sigma > 0$ .
- (a) Specify and solve the household problem.
  - (b) Assuming that the risk-free asset is in zero net supply and the risky asset is in positive net supply, find the equilibrium rates of return for the risk free and the risky asset.
  - (c) Discuss and interpret your results. Find the equilibrium stochastic discount factor which is implicit in this economy.
3. Consider a limited participation finance market economy in which both types of households, participating and non-participating households, have a log utility function. The risky asset entitles the holder to receive a dividend that follows the process  $dD(t) = \sigma D(t)dW(t)$ .
- (a) Specify and solve the household problem.
  - (b) Assuming that the risk-free asset is in zero net supply and the risky asset is in positive net supply, find the equilibrium rates of return for the risk free and the risky asset.
  - (c) Discuss and interpret your results. Find the equilibrium stochastic discount factor which is implicit in this economy.
4. Consider a frictionless finance market economy in which there are two assets, one risky asset and one risk free asset. Agents are heterogeneous as regards their preferences, although they can be divided into two groups:  $L$  agents have a logarithmic utility function and  $H$  agents have a CRRA utility function with coefficient of risk aversion equal to 2. The risky asset entitles the holder to receive a dividend that follows the process  $dD(t) = \sigma D(t)dW(t)$ .
- (a) Specify and solve the households' problems.
  - (b) Assuming that the risk-free asset is in zero net supply and the risky asset is in positive net supply write the equations allowing for obtaining the equilibrium rates of return for the risk free and the risky asset.
  - (c) Discuss and interpret your results.
5. Assume a finance economy in which there are two assets and no frictions, and that the household problem is

$$\max_{C,w} \int_0^{\infty} C(t)e^{-\rho t} dt$$

subject to

$$dN(t) = [r(t) + (\tilde{\mu}_s(t) - r(t))w(t)]N(t)dt - C(t)dt + \sigma_s w(t)N(t)dW(t)$$

where  $C$ ,  $N$ ,  $w$  denote consumption, net wealth, the weight of the risky asset in net wealth and  $r$  is the risk free interest rate and rate of return for the risky asset follows the process  $dr_s(t) = (\mu_s(t) + q(t)^{-1})dt + \sigma_s(t)dW(t)$ . The dividend follows the process  $dD(t) = D(t)(gdt + \sigma dW(t))$  and  $q$  is the price-dividend ratio.

- (a) Write the Hamilton-Jacobin-Bellman equation. Find necessary conditions for the existence of general equilibrium. Prove that the solution for consumption and portfolio weights are undetermined
  - (b) Define and find the general equilibrium processes for consumption and the rates of return.
  - (c) Provide an intuition for your results.
6. Consider an economy in which there is technological illiquidity, agents are homogeneous, there are two assets, one risk-free and one risky asset, agents have no constraints in their portfolio investments and have logarithmic preferences. Assume that the gross investment function is  $\Phi(i) = \iota^\alpha$ , where  $\iota$  is investment expenditure and  $0 < \alpha < 1$  and the production function is linear  $Y = AK(t)$ . In this economy the risk free asset is in zero net supply and the risky asset is in positive net supply.
- (a) Define and write the equations for the dynamic general equilibrium of this economy.
  - (b) Find the equilibrium rates of return for the risk free and the risky asset.
  - (c) Discuss and interpret your results.
7. Consider an economy in which there is technological illiquidity and there are two assets, one risk-free and one risky asset. Households have homogenous preferences, have logarithmic preferences, but there is one group of agents which does not participate in the risky asset market. Assume that the gross investment function is  $\Phi(i) = \iota^\alpha$ , where  $\iota$  is investment expenditure and  $0 < \alpha < 1$  and the production function is linear  $Y = AK(t)$ . In this economy the risk free asset is in zero net supply and the risky asset is in positive net supply.
- (a) Define and write down the equations for the dynamic general equilibrium of this economy.
  - (b) Find the equilibrium rates of return for the risk free and the risky asset.
  - (c) Discuss and interpret your results.