Advanced macroeconomics 2021-2022
Problem set 4: New-Keynesian macro models

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1 General questions

1. Consider the benchmark DGE model in which there is with capital accumulation and competitive markets and an analogous New-Keynesian model in which there is perfect competition in the markets for the final good and the production factors, but there is monopolistic competition in the intermediate good industries, assuming that the number of varieties and there is a monopoly in every industry.
   (a) Compare the productive structures
   (b) Provide definitions of the general equilibrium
   (c) Which type of markup one would expect and what are the consequences of the existence of that markup to the characterization of the general equilibrium (both as regards its transition dynamics and long-run properties).
2. Consider the a new-Keynesian model with Cournotian monopolistic competition with entry of new firms. In this model, differently for the case in which there is monopolistic competition without entry, the markup becomes endogenous.
   (a) Discuss which assumptions you need to introduce in the model in order to have endogenous markups.
   (b) What are the consequences, for the dynamic features of the general equilibrium, of the existence of endogenous markups ?

2 Problems

1. Consider a "love for varieties model" in a new-Keynesian model with capital accumulation. The household optimizes for the bundle of consumption goods $\Big(c(j,t)\Big)_{j\in[0,1]}$ with varieties $j \in [0,1]$ and wealth accumulation by solving a two-stage problem: in

the first stage they solve the problem

$$\begin{aligned}&\min_{(c(j,t))_{j\in[0,1]}} \int_0^1 p(j,t)\,c(j,t)\,dj\\&\text{subject to}\\&\Big(\int_0^1 c(j,t)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\Big)^{\frac{\varepsilon}{\varepsilon-1}} = C(t)\end{aligned}$$

where $p(j,t)$ is the price of variety j, and $C(t)$ is an index of consumption $C(t)$, and in the second stage they solve the problem

$$\begin{aligned}&\max_{C(\cdot)} \int_0^\infty \ln(C(t))\,e^{-\rho t}\,dt\\&\text{subject to}\\&\dot{W} = r(t)\,W(t) + \omega(t)\,\ell(t) - C(t)\\&W(0) = W_0 \text{ given}\end{aligned}$$

where W denotes financial wealth, r is the gross real rate of return, ω is the wage rate and ℓ is the labor effort, which is supplied inelastically. The producer of variety j problem, is[1]

$$\begin{aligned}&\max_{\ell(j,t),i(j,t)} \int_0^\infty \Big((A\,k(j,t)^{\alpha}\,\ell(j,t)^{1-\alpha})^{1-\mu}\,C(t)^{\mu} - w(t)\,\ell(j,t) - i(j,t)\Big)\,e^{-R(t)}\,dt\\&\text{subject to}\\&\frac{dk(j,t)}{dt} = i(j,t) - \delta\,k(j,t)\\&k(j,0) = k_{j,0} \text{ given.}\end{aligned}$$

(a) Define the general equilibrium (DGE) for this economy. Assume that the aggregator for any variable X, $X(t) = \Big(\in_0^1 x(j,t)^{1-\mu}\,dj\Big)^{\frac{1}{1-\mu}}$ and that the total supply of labor is $L(t) = 1$.

(b) Prove that the solution of the consumer first-stage problem is $c(j,t) = \Big(\frac{p(j,t)}{P(t)}\Big)^{-\varepsilon} C(t)$, together with the constraint $P(t) = \Big(\int_0^1 p(j,t)^{1-\varepsilon}\,dj\Big)^{\frac{1}{1-\varepsilon}}$. Find the first-order conditions for the second-stage problem

(c) Assuming that the production function of variety j is $y(j,t) = A\,k(j,t)^{\alpha}\,\ell(j,t)^{1-\alpha}$, show that the sale of a monopolist firm producing variety j is $s(j,t) = p(j,t)\,y(j,t) = (A\,k(j,t)^{\alpha}\,\ell(j,t)^{1-\alpha})^{1-\mu}\,C(t)^{\mu}$ where $\mu = 1/\varepsilon$.

[1] See section 2.1.3 in `https://pmbbrito.github.io/cursos/phd/am/am2122_NKRamsey.pdf`.

(d) Solve the problem of the problem for the firm in industry j. Show that the solution is symmetric across varieties and that

$$\ell(j,t) = \frac{(1-\alpha)\,(1-\mu)\,s(t)}{w(t)},\ k(j,t) = \frac{\alpha\,(1-\mu)\,s(t)}{(r(t)+\delta},\ y(j,t)^{1-\mu}c(t)^{\mu} = s(t)$$

where $s(t)$ are the optimal sales

$$s(t) = \Big[A\,(1-\mu)\,\Big(\frac{\alpha}{r(t)+\delta}\Big)^{\alpha}\,\Big(\frac{1-\alpha}{w(t)}\Big)^{1-\alpha}\Big)^{1-\mu}\,C(t)^{\mu}$$

(e) Prove that the DGE for this economy is represented by the dynamic system

$$\begin{aligned}\dot{K} &= (1-\mu)\,AK^{\alpha} - \delta\,K - C\\ \dot{C} &= C\,\Big((1-\mu)\,AK^{\alpha-1} - \delta - \rho\Big)\\ K(0) &= k_0 \text{ given}\end{aligned}$$

and a transversality condition. Compare the dynamics and the assymptotics of the model with the benchmark DGE model with perfect competition.

2. Consider a version of the Benhabib and Farmer (1994) model in which the production function at the private level is $y = Ak^{\alpha}\,\ell^{b}$ and there is a externality which is a function of the aggregate labor $A(L) = L^{\beta-b}$, where $\beta > b > 0$ and $\alpha > 0$. The household problem is

$$\begin{aligned}&\max_{c(\cdot),\ell(\cdot)} \int_0 \infty\Big(\frac{c(t)^{1-\theta}-1}{1-\theta} - \frac{\ell(t)^{1+\xi}}{1+\xi}\Big)\,e^{-\rho\,t}\,dt\\ &\text{subject to}\\ &\dot{k} = Ak^{\alpha}\,\ell^{b} - c\\ &k(0) = k_0 \text{ given.}\end{aligned}$$

All the parameters are positive constants.

(a) Define the dynamic general equilibrium (DGE).

(b) Find the optimality condition for the household.

(c) Prove that, after considering the micro-macro consistency conditions, the DGE is the solution of the system

$$\begin{aligned}\dot{K} &= K^{\alpha}\,L^{\beta} - C(K,L)\\ \dot{L} &= \frac{L}{\xi-\beta}\Big(\alpha\,\frac{C(K,L)}{K} - \rho\Big)\end{aligned}$$

where $C(K,L) = b^{\frac{1}{\theta}}\,L^{\frac{1-\beta}{\theta}}\,K^{\frac{\alpha}{\theta}}$.

(d) Assume that $0 < \beta < \xi < \beta\,(1+\theta)$. Draw the phase diagram. Find the steady state and study its local dynamic properties.

(e) Provide an intuition to the results that you have obtained.

References

Benhabib, J. and Farmer, R. (1994). Indeterminacy and increasing returns. *Journal of Economic Theory*, 63:19–41.