Advanced macroeconomics 2022-2023
Problem set 2: Ramsey and DGE models

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1 Ramsey: general

1 Discuss the effects of assuming decreasing marginal returns to capital and intertemporal substitution in consumption on the dynamics of the Ramsey model.

2* Discuss the effects of assuming anticipated permanent productivity shocks on the optimal path of the economy, according to the Ramsey model[1].

3* Discuss the effects of effects of non-anticipated temporary productivity shocks on the optimal path of the economy, according to the Ramsey model

2 Ramsey: Exogenous labor

1 Consider a Ramsey model in which there is depreciation of capital, and the production function is Cobb-Douglas $y = f(k) \equiv A\,k^{\alpha}$, for $A > 0$ and $0 < \alpha < 1$. That is the capital accumulation constraint is $\dot{k} = Ak^{\alpha} - c - \delta\,k$, where $\delta > 0$, where all the variables are in per-capita terms. The instantaneous utility function is $u(c) = \log(c)$ and the rate of time preference is $\rho > 0$.

(a) Solve the Ramsey problem by using the PMP.

(b) Draw the phase diagram.

(c) In this model there is a manifold passing through two steady states, one in which both k and c are positive, and another one in which $c = 0$ and $f(k) = \delta k$ for $k > 0$. Prove that there are admissible trajectories connecting those two points. Explain why those trajectories cannot be optimal.

(d) Perform a comparative dynamics exercise for an increase in δ. Provide an intuition for your results.

[1]Questions are marked with asterisks depending on their degree of difficulty.

2 Consider a Ramsey model in which unfunded government expenditures can exist. The economy's resource constraint is $\dot{k} = Ak^{\alpha} - c - g - \delta\, k$, where $g \geq 0$ is a public transfer, $\delta > 0$, $A > 0$, and $0 < \alpha < 1$. The instantaneous utility function is isoelastic $u(c) = \dfrac{c^{1-\theta} - 1}{1-\theta}$, for $\theta > 0$, and the rate of time preference is $\rho > 0$.

(a) Solve the Ramsey problem by using the PMP.

(b) Is it possible to solve explicitly the Ramsey using the DPP ?

(c) Perform a comparative dynamics exercise for an increase in g. Provide one intuition for your results.

3 Ramsey: Endogenous labor

1 Consider the following Ramsey model with endogenous labor supply

$$\max_{c(\cdot),\ell(\cdot)} \int_0^{\infty} \Big(\ln\left(c(t)\right) - \psi\,\ell \Big)\, e^{-\rho}\, dt$$
$$\text{subject to}$$
$$\dot{k} = A\, k^{\alpha}\, \ell^{1-\alpha}$$
$$k(0) = k_0 > 0 \text{ given}$$
$$\lim_{t\to\infty} k(t) \geq 0$$

(a) Find the first order conditions for optimality.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant increase in TFP A. Provide one intuition for your results.

2* Consider a Ramsey model with endogenous labour with additively separable preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{c^{1-\theta}-1}{1-\theta} - \psi\, \frac{\ell^{1+\zeta}}{1+\zeta},\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

in which the rate of time preference is $\rho > 0$, the production function is

$$f(k,\ell) = A\, k^{\alpha}\, \ell^{1-\alpha},\ \text{for } A > 0,\ 0 < \alpha < 1,$$

and there is no capital depreciation.

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

3* Consider a Ramsey model with endogenous labour with KPR preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{\Big(c\left(1-\psi\,\ell^{\eta}\right)\Big)^{1-\theta} - 1}{1-\theta}, \text{ for } \theta > 0,\ \psi > 0,\ \eta > 0$$

in which the rate of time preference is $\rho > 0$, the production function is

$$f(k,\ell) = A\,k^{\alpha}\,\ell^{1-\alpha}, \text{ for } A > 0,\ 0 < \alpha < 1,$$

and there is no capital depreciation.

(a) Write the MHDS

(b) Build the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

4* Consider a Ramsey model with endogenous labour with GHH preferences and Cobb-Douglas technology. That is

$$u(c,\ell) = \frac{1}{1-\theta}\left(\left(c - \psi\,\frac{\ell^{1+\zeta}}{1+\zeta}\right)^{1-\theta} - 1\right),\ \theta > 0,\ \psi > 0,\ \zeta > 0$$

and

$$f(k,\ell) = A\,k^{\alpha}\,\ell^{1-\ell}$$

(a) Write the MHDS

(b) Draw the phase diagram

(c) Study the effects of a non-anticipated, permanent and constant shocks in A, ψ and ρ. Provide one intuition for your results.

5* Consider a Ramsey model in which labor is endogenous. The constraint of the economy is given by equation $\dot{k} \equiv \dfrac{dk(t)}{dt} = k^{\alpha}\,\ell^{1-\alpha} - c$, where $0 < \alpha < 1$. The central planner optimal allocation is a solution to the HJB equation

$$\rho\, v(k) = \max_{c,\ell} \Big\{ \ln(c) - \psi\,\ell + v'(k)\left(k^{\alpha}\,\ell^{1-\alpha} - c\right)\Big\}.$$

where $\psi > 0$, and $\rho > 0$, with the usual interpretation: k, ℓ and c denote the per-capital capital stock, labor effort, and consumption, respectively, and $v(\cdot)$ is the value function.

(a) Find the optimal policy functions and write the optimal HJB equation.

(b) Using the envelope theorem, and the policy functions you obtained in (a), we can obtain a representation of the optimality conditions as a dynamic system in (k, c),

$$\dot{k} = \frac{R(c)}{\alpha}\,k - c$$
$$\dot{c} = c\left(R(c) - \rho\right),$$

where $R(c) \equiv \alpha \left(\dfrac{1-\alpha}{\psi\, c}\right)^{\frac{1-\alpha}{\alpha}}$. Prove this.

(c) Draw the phase diagram, assuming that k is a pre-determined variable, and the optimal path is conditionally stable. Explain your reasoning.

(d) Study the effects of non-anticipated, permanent, and constant increases in ψ and in ρ. Provide economic intuitions for your results.

4 DGE: exogenous labor

1 Consider a DGE economy in which the utility function is $u(c) = \log(c)$, the rate of time preference is $\rho > 0$, there is a constant number of households N, there is no unemployment, and the technology for firms is CES

$$Y = A\left(\alpha\, K^{\eta} + (1-\alpha)\, L^{\eta}\right)^{\frac{1}{\eta}}$$

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant shock in A. Provide an intuition for your results.

2 Consider a DGE economy in which the utility function is $u(c) = \log(c)$ and the production function is Cobb-Douglas in which the government raises an income tax and has a budget balanced fiscal policy. Denoting per capita government expenditure by g and the tax rate is denoted by τ and both are constant through time. The budget balance rule is $g = \tau\left(r(t)a(t) + w(t)\right)$. Assume that households supply labor inelastically and they have the budget constraint $\dot{a} = (1-\tau)\left(r(t)a(t) + w(t)\right) - c(t) + g(t)$.

(a) Define the dynamic general equilibrium and provide the dynamic system allowing for the determination of the DGE.

(b) Build the phase diagram.

(c) Study the effects of a non-anticipated, permanent and constant increase in g. Provide one intuition for your results.