Advanced macroeconomics 2022-2023
Problem set 3: New-Keynesian macro models

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1 General questions

1. Consider the benchmark DGE model in which there is with capital accumulation and competitive markets and an analogous New-Keynesian model in which there is perfect competition in the markets for the final good and the production factors, but there is monopolistic competition in the intermediate good industries, assuming that the number of varieties and there is a monopoly in every industry.
   (a) Compare the productive structures
   (b) Provide definitions of the general equilibrium
   (c) Which type of markup one would expect and what are the consequences of the existence of that markup to the characterization of the general equilibrium (both as regards its transition dynamics and long-run properties).
2. Consider the a new-Keynesian model with Cournotian monopolistic competition with entry of new firms. In this model, differently for the case in which there is monopolistic competition without entry, the markup becomes endogenous.
   (a) Discuss which assumptions you need to introduce in the model in order to have endogenous markups.
   (b) What are the consequences, for the dynamic features of the general equilibrium, of the existence of endogenous markups ?

2 Monopolistic competition models

1 Consider a "love for varieties model" in a new-Keynesian model with capital accumulation. The household optimizes for the bundle of consumption goods $\Big(c(j,t)\Big)_{j\in[0,1]}$ with varieties $j \in [0,1]$ and wealth accumulation by solving a two-stage problem: in the first stage they solve the problem

$$\min_{(c(j,t))_{j\in[0,1]}} \int_0^1 p(j,t)\, c(j,t)\, dj$$
$$\text{subject to}$$
$$\Big(\int_0^1 c(j,t)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\Big)^{\frac{\varepsilon}{\varepsilon-1}} = C(t)$$

where $p(j,t)$ is the price of variety j, and $C(t)$ is an index of consumption $C(t)$, and in the second stage they solve the problem

$$\max_{C(\cdot)} \int_0^\infty \ln(C(t))\, e^{-\rho t}\, dt$$
$$\text{subject to}$$
$$\dot{W} = r(t)\, W(t) + \omega(t)\, \ell(t) - C(t)$$
$$W(0) = W_0 \text{ given}$$

where W denotes financial wealth, r is the gross real rate of return, ω is the wage rate and ℓ is the labor effort, which is supplied inelastically. The producer of variety j problem, is[1]

$$\max_{\ell(j,t),i(j,t)} \int_0^\infty \Big((A\, k(j,t)^\alpha\, \ell(j,t)^{1-\alpha})^{1-\mu}\, C(t)^\mu - w(t)\, \ell(j,t) - i(j,t)\Big)\, e^{-R(t)}\, dt$$
$$\text{subject to}$$
$$\frac{dk(j,t)}{dt} = i(j,t) - \delta\, k(j,t)$$
$$k(j,0) = k_{j,0} \text{ given.}$$

(a) Define the general equilibrium (DGE) for this economy. Assume that the aggregator for any variable X, $X(t) = \Big(\in_0^1 x(j,t)^{1-\mu}\, dj\Big)^{\frac{1}{1-\mu}}$ and that the total supply of labor is $L(t) = 1$.

(b) Prove that the solution of the consumer first-stage problem is $c(j,t) = \Big(\dfrac{p(j,t)}{P(t)}\Big)^{-\varepsilon} C(t)$, together with the constraint $P(t) = \Big(\int_0^1 p(j,t)^{1-\varepsilon}\, dj\Big)^{\frac{1}{1-\varepsilon}}$. Find the first-order conditions for the second-stage problem

(c) Assuming that the production function of variety j is $y(j,t) = A\, k(j,t)^\alpha\, \ell(j,t)^{1-\alpha}$, show that the sale of a monopolist firm producing variety j is $s(j,t) = p(j,t)\, y(j,t) = (A\, k(j,t)^\alpha\, \ell(j,t)^{1-\alpha})^{1-\mu}\, C(t)^\mu$ where $\mu = 1/\varepsilon$.

[1] See section 2.1.3 in `https://pmbbrito.github.io/cursos/phd/am/am2223_NKRamsey.pdf`.

(d) Solve the problem of the problem for the firm in industry j. Show that the solution is symmetric across varieties and that

$$\ell(j,t) = \frac{(1-\alpha)\,(1-\mu)\,s(t)}{w(t)},\; k(j,t) = \frac{\alpha\,(1-\mu)\,s(t)}{(r(t)+\delta},\; y(j,t)^{1-\mu} c(t)^{\mu} = s(t)$$

where $s(t)$ are the optimal sales

$$s(t) = \left[A\,(1-\mu)\left(\frac{\alpha}{r(t)+\delta}\right)^{\alpha}\left(\frac{1-\alpha}{w(t)}\right)^{1-\alpha}\right)^{1-\mu} C(t)^{\mu}$$

(e) Prove that the DGE for this economy is represented by the dynamic system

$$\begin{aligned}
\dot{K} &= (1-\mu)\,AK^{\alpha} - \delta\,K - C \\
\dot{C} &= C\left((1-\mu)\,AK^{\alpha-1} - \delta - \rho\right) \\
K(0) &= k_0 \text{ given}
\end{aligned}$$

and a transversality condition. Compare the dynamics and the assymptotics of the model with the benchmark DGE model with perfect competition.

2 Consider a new-Keynesian model with monopolistic competition. The final product market is competitive and the technology of production uses a continuum of intermediate good varieties $j \in [0,1]$. Each variety is produced by a monopolist and uses capital as the only input. In the next questions, items (a) to (c) are dedicated to building up the details for the definition of the general equilibrium and items (d) to (f) are concerned with its representation and characterization.

(a) Each representative household solves the following problem

$$\max_{c(\cdot)} \int_0^{\infty} \ln c(t)\, e^{-\rho\, t}\, dt, \text{ subject to } \dot{w} = r(t)\, w(t) - c(t)$$

for $\rho > 0$, $w(0) = w_0$ fixed, and $\lim_{t\to\infty} e^{-R(t)}\, w(t) \geq 0$, with $R(t) = \int_0^t r(s)\, ds$. The variables $c(t)$, $w(t)$, and $r(t)$ denote consumption, net asset position, and the asset market rate of return, respectively. Find the optimality conditions.

(b) The problem of the producer of the final good is

$$\max_{\mathbf{x}(t)} \left\{ y(t) - \int_0^1 p(j,t)\, x(j,t)\, dj, \text{ subject to } F(\mathbf{x}(t)) = y(t) \right\}$$

where $F(\mathbf{x}(t))$, and $y(t)$ denote the production function, and the output at time t. The production function $F(\mathbf{x}(t)) = \left(\int_0^1 A\, x(j,t)^{1-\mu}\, dj\right)^{\frac{1}{1-\mu}}$, where A is a productivity parameter, and $0 < \mu < 1$. Prove that the solution is $x^*(j,t) = \left(\frac{p(j,t)}{A}\right)^{-\frac{1}{\mu}} y(t)$.

(c) Assuming that the production function for the monopolist producing variety j is $x(j,t) = k(j,t)$, where $k(j,t)$ is the stock of capital, and that the cost per unit of investment is ξ, the j-monopolist problem is

$$\max_{k(j,\cdot)} \int_0^{\infty} \Big(\pi(j,t) - \xi\, \dot{k}(j,t)\Big)\, e^{-R(t)}\, dt \text{ given } k(j,0) = k(0)$$

where $R(t) = \int_0^t r(s)\, ds$ and $r(t)$ are the discount rate and the asset market rate of return at time t. Show that the profit is $\pi(j,t) = A\, k(j,t)^{1-\mu}\, y(t)^{\mu}$. Prove that the optimality condition for the j-monopolist is

$$(1-\mu)\, Ak(j,t)^{-\mu} y(t)^{\mu} = \xi\, r(t) \text{ for each } t \in [0,\infty).$$

(d) Define the DGE (dynamic general equilibrium) for this economy. Show that it can be characterized by the problem,

$$\begin{aligned} \dot{k} &= \bar{r}\, k - c, \text{ where } \bar{r} = \frac{1-\mu}{\xi}\, A^{\frac{1}{1-\mu}} \\ \dot{c} &= c\,(\bar{r} - \rho) \\ k(0) &= k_0 \text{ fixed} \\ \lim_{t\to\infty} &\frac{k(t)}{c(t)}\, e^{-\rho\, t} = 0 \end{aligned}$$

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(e) Characterize the equilibrium dynamics for $\bar{r} = \rho$ and for $\bar{r} > \rho$.

(f) Starting from $\bar{r} = \rho$, determine the dynamic effects from a permanent and non-anticipated increase in the productivity, A.

3 Externalities

1 Consider a version of the **?** model in which the production function at the private level is $y = Ak^{\alpha}\, \ell^{b}$ and there is a externality which is a function of the aggregate labor $A(L) = L^{\beta-b}$, where $\beta > b > 0$ and $\alpha > 0$. The household problem is

$$\max_{c(\cdot),\ell(\cdot)} \int_0^{\infty} \Big(\frac{c(t)^{1-\theta}-1}{1-\theta} - \frac{\ell(t)^{1+\xi}}{1+\xi}\Big)\, e^{-\rho\, t}\, dt$$

$$\begin{aligned} &\text{subject to} \\ &\dot{k} = Ak^{\alpha}\, \ell^{b} - c \\ &k(0) = k_0 \text{ given.} \end{aligned}$$

All the parameters are positive constants.

(a) Define the dynamic general equilibrium (DGE).

(b) Find the optimality condition for the household.

(c) Prove that, after considering the micro-macro consistency conditions, the DGE is the solution of the system

$$\dot{K} = K^{\alpha} L^{\beta} - C(K, L)$$
$$\dot{L} = \frac{L}{\xi - \beta}\Big(\alpha \frac{C(K, L)}{K} - \rho\Big)$$

where $C(K, L) = b^{\frac{1}{\theta}} L^{\frac{1-\beta}{\theta}} K^{\frac{\alpha}{\theta}}$.

(d) Assume that $0 < \beta < \xi < \beta(1+\theta)$. Draw the phase diagram. Find the steady state and study its local dynamic properties.

(e) Provide an intuition to the results that you have obtained.