Advanced Mathematical Economics
(Economia Matemática Avançada)
PhD in Economics
Universidade de Lisboa, ISEG

Paulo Brito
pbrito@iseg.ulisboa.pt

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Goals

- Study of functional equations in economics, and, in particular, dynamic systems in the continuum. We will study, at an intermediate level, differential equations (ordinary, partial and stochastic), optimisation of functionals, optimal control of differential equations (ordinary, partial and stochastic) and (possibly) dynamic game theory.
- Application to some economic theory models from growth theory, dynamic general equilibrium, age-structured models, dynamics of distribution, finance, stochastic dynamics, and dynamic games.
- If we have time, we will also cover some special topics: models with singularities and models with thresholds.

Approach

- We will follow a heuristic approach: emphasise the characterisation of the dynamic properties generated by several dynamic systems rather than trying to prove existence and uniqueness of solutions to the generic functional equations. This is not a course on functional analysis.
- We will supply study material: as a minimum requirement we will be made available notes for every topic. They do not substitute studying from other literature (referenced or not). A problem set for every topic will also be handed.
- All material will be posted at `https://pmbbrito.github.io/cursos/phd/ame/ame_1920.html`. After its initial posting the class notes and the problem sets may be changed along the semester. **Warning: please check the date of the document before downloading**.

Assumed background

- Ideally: mathematics and economic theory at the level of the Masters in Economics, Monetary and Financial Economics, Quantitative Finance (ISEG)
- At least: calculus, algebra, optimisation and probability theory, at an intermediate level.

Topics covered

The main topics which will be covered are the following:

- Ordinary differential equations (ODE)
- Optimal control of ordinary differential equations (OC-ODE)
- Partial differential equations (PDE)
- Optimal control of PDE (OC-PDE)
- Stochastic differential equations (SDE)
- Optimal control of SDE (OC-SDE)

Bibliography

General textbooks covering the topics which will be lectured are:

- ODE: Guckenheimer and Holmes (1990), Hale and Koçak (1991), Perko (1996)
- OC-ODE: Kamien and Schwartz (1991), Grass et al. (2008), Weber (2011)
- PDE: Evans (2010), Olver (2014), , Salsa (2016)
- SDE: Øksendal (2003), Pavliotis (2014)
- OC-SDE: Fleming and Rishel (1975),

Textbooks in macroeconomics and growth theory:

- Growth theory: Acemoglu (2009)
- Macroeconomics: Heijdra (2009), Ljungqvist and Sargent (2012)
- Macro-finance: Stokey (2009)

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.

Evans, L. C. (2010). *Partial Differential Equations*, volume 19 of *Graduate Series in Mathematics*. American Mathematical Society, Providence, Rhode Island, second edition.

Fleming, W. H. and Rishel, R. W. (1975). *Deterministic and Stochastic Optimal Control*. Springer-Verlag.

Grass, D., Caulkins, J. P., Feichtinger, G., Tragler, G., and Behrens, D. A. (2008). *Optimal Control of Nonlinear Processes. With Applications in Drugs, Corruption, and Terror*. Springer.

Guckenheimer, J. and Holmes, P. (1990). *Nonlinear Oscillations and Bifurcations of Vector Fields*. Springer-Verlag, 2nd edition.

Hale, J. and Koçak, H. (1991). *Dynamics and Bifurcations*. Springer-Verlag.

Heijdra, B. J. (2009). *Foundations of Modern Macroeconomics*. Oxford University Press, 2 edition.

Kamien, M. I. and Schwartz, N. L. (1991). *Dynamic optimization, 2nd ed.* North-Holland.

Ljungqvist, L. and Sargent, T. J. (2012). *Recursive Macroeconomic Theory*. MIT Press, Cambridge and London, 3rd edition.

Øksendal, B. (2003). *Stochastic Differential Equations*. Springer, 6th edition.

Olver, P. J. (2014). *Introduction to Partial Differential Equations*. Undergraduate Texts in Mathematics. Springer International Publishing, 1 edition.

Pavliotis, G. A. (2014). *Stochastic Processes and Applications: Diffusion Processes, the Fokker-Planck and Langevin Equations*. Texts in Applied Mathematics 60. Springer-Verlag New York, 1 edition.

Perko, L. (1996). *Differential Equations and Dynamical Systems, 2nd Ed.* Springer-Verlag.

Salsa, S. (2016). *Partial Differential Equations in Action: From Modelling to Theory*. Number 99 in Unitext. Springer International Publishing, third edition.

Stokey, N. L. (2009). *The Economics of Inaction*. Princeton.

Weber, T. A. (2011). *Optimal Control Theory with Applications in Economics*. The MIT Press.

Other references will be given along the way and would be cited in the classnotes.

Assessment

The assessment will be made by a final written closed book exam (see : https://aquila.iseg.utl.pt/aquila/getFile.do?method=getFile&fileId=296795regulamento da avaliação dos doutoramentos do ISEG). The questions will be taken from, or will be similar, to the ones included in the problem sets.

Sessions

Tentative scheduling of sessions:

session	date	session	syllabus
1	18/02/2020	20:00 - 22:00	Presentation. Introduction.
2	03/03/2020	20:00 - 22:00	ODE: linear
3	10/03/2020	20:00 - 22:00	ODE: non-linear - normal forms and bifurcations
4	17/03/2020	20:00 - 22:00	ODE: non-linear - non-smooth and singular
5	24/03/2020	20:00 - 22:00	ODE: applications
6	. 31/03/2020	20:00 - 22:00	OC-ODE: CV, PMP, DP
7	14/04/2020	20:00 - 22:00	OC-ODE: extensions and applications
8	21/04/2020	20:00 - 22:00	PDE: first-order
9	28/04/2020	20:00 - 22:00	OC-PDE: first-order
10	05/05/2020	20:00 - 22:00	PDE: parabolic, OC-PDE: parabolic
11	12/05/2020	20:00 - 22:00	SDE: linear diffusion equations. OC-SDE
12	19/05/2020	20:00 - 22:00	PDE and SDE: applications

Software

Although this is not a course in numerical methods, the use of computers helps a lot in illustrating the solutions, solving, studying the dynamic properties, and estimating the models.

Useful software for solving differential equations:

- public license:
  - specialized for ODE's: auto (http://indy.cs.concordia.ca/auto/, http://www.dam.brown.edu/people/sandsted/homcont.php), and xpp (http://www.math.pitt.edu/~bard/xpp/xpp.html)
  - generic languages: python (https://www.python.org/ and for ODE's https://docs.scipy.org/doc/scipy/reference/integrate.html), R (https://www.r-project.org/ and an example for solving ODE's https://cran.r-project.org/web/packages/sundialr/vignettes/my-vignette.html or https://cran.r-project.org/web/packages/deSolve/index.html), sagemath (http://www.sagemath.org/)

- proprietary: Mathematica (`https://www.wolfram.com/mathematica/`), Maple (`https://maplesoft.com/`, Matlab (`https://www.mathworks.com/products/matlab.html`).