Advanced Mathematical Economics
(Economia Matemática Avançada)
PhD in Economics
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Goals

- Study of functional equations in economics, and, in particular, dynamic systems in the continuum. We will study, at an intermediate level, differential equations (ordinary, partial and stochastic), optimisation of functionals, optimal control of differential equations (ordinary, partial and stochastic) and (possibly) dynamic game theory.
- Application to some economic theory models from growth theory, dynamic general equilibrium, age-structured models, dynamics of distribution, finance, stochastic dynamics, and dynamic games.
- If we have time, we will also cover some special topics: models with singularities and models with thresholds.

Approach

- We will follow a heuristic approach: emphasise the characterisation of the dynamic properties generated by several dynamic systems rather than trying to prove existence and uniqueness of solutions to the generic functional equations. This is not a course on functional analysis.
- We will supply study material: as a minimum requirement we will be made available notes for every topic. They do not substitute studying from other literature (referenced or not). A problem set for every topic will also be handed.
- All material will be posted at `https://pmbbrito.github.io/cursos/phd/ame/ame_2021.html`. After its initial posting the class notes and the problem sets may be changed along the semester. **Warning: please check the date of the document before downloading it**.

Assumed background

- Ideally: mathematics and economic theory at the level of the Masters in Economics, Monetary and Financial Economics, Quantitative Finance (ISEG)
- At least: calculus, algebra, optimisation and probability theory, at an intermediate level.

Topics covered

The main topics which will be covered are the following:

- Ordinary differential equations (ODE)
- Optimal control of ordinary differential equations (OC-ODE)
- Partial differential equations (PDE): first-order and parabolic
- Optimal control of PDE (OC-PDE): of first order and parabolic PDE's
- Stochastic differential equations (SDE)
- Optimal control of SDE (OC-SDE)

Distributed material

- **Lecture notes**: will be posted close to the day of the lecture (see last year webpage for previous versions of the notes)
- **Problem sets**: with applications of the material taught. **Disclaimer**: these are seen as problems allowing for the application of the subjects which are lectured. There is no certainty that they are well posed or interesting. Solutions will not be provided, although I am ready to clarify **specific** points of their solution.

Assessment

The assessment will be made by a final written open book (no electronic device consultation) exam (see : `https://aquila.iseg.utl.pt/aquila/getFile.do?method=getFile&fileId=296795`regulamento da avaliação dos doutoramentos do ISEG). The questions will be taken from, or will be similar, to the ones included in the problem sets.

Sessions

Tentative scheduling of sessions:

session	date	session	syllabus
1	16/09/2020	18:00 - 20:00	Presentation. Introduction.
2	23/09/2020	18:00 - 20:00	ODE: linear
3	30/09/2020	18:00 - 20:00	ODE: linear
4	14/10/2020	18:00 - 20:00	ODE: non-linear
5	21/10/2020	18:00 - 20:00	ODE: applications
6	. 28/10/2020	18:00 - 20:00	OC-ODE: CV, PMP, DP
7	04/11/2020	18:00 - 20:00	OC-ODE: extensions and applications
8	11/11/2020	18:00 - 20:00	PDE: first-order
9	18/11/2020	18:00 - 20:00	OC-PDE: first-order
10	25/11/2020	18:00 - 20:00	PDE: parabolic,
11	02/12/2020	18:00 - 20:00	OC-PDE: parabolic
12	09/12/2020	18:00 - 20:00	SDE
13	16/12/2020	18:00 - 20:00	OC-SDE

Bibliography

General textbooks covering the topics which will be lectured are:

- ODE: Guckenheimer and Holmes (1990), Hale and Koçak (1991), Perko (1996)
- OC-ODE: Kamien and Schwartz (1991), Grass et al. (2008), Weber (2011)
- PDE: Evans (2010), Olver (2014), Kuehn (2019), Salsa (2016)
- SDE: Øksendal (2003), Pavliotis (2014)
- OC-SDE: Pham (2009), Seierstad (2009), Fleming and Rishel (1975),

Textbooks in continous-time applications to economics:

- Growth theory: Acemoglu (2009)
- Macroeconomics: Heijdra (2009), Turnovsky (1995), Brock and Malliaris (1989)
- Macro-finance: Stokey (2009)
- Finance: Cvitanić and Zapatero (2004)
- Microeconomics: Cvitanić and Zhang (2013)
- Game theory: Dockner et al. (2000)

References

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.

Brock, W. A. and Malliaris, A. G. (1989). *Differential Equations, Stability and Chaos in Dynamic Economics*. North-Holland.

Cvitanić, J. and Zapatero, F. (2004). *Introduction to the Economics and Mathematics of Financial Markets*. MIT Press.

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Evans, L. C. (2010). *Partial Differential Equations*, volume 19 of *Graduate Series in Mathematics*. American Mathematical Society, Providence, Rhode Island, second edition.

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Grass, D., Caulkins, J. P., Feichtinger, G., Tragler, G., and Behrens, D. A. (2008). *Optimal Control of Nonlinear Processes. With Applications in Drugs, Corruption, and Terror*. Springer.

Guckenheimer, J. and Holmes, P. (1990). *Nonlinear Oscillations and Bifurcations of Vector Fields*. Springer-Verlag, 2nd edition.

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Heijdra, B. J. (2009). *Foundations of Modern Macroeconomics*. Oxford University Press, 2 edition.

Kamien, M. I. and Schwartz, N. L. (1991). *Dynamic optimization, 2nd ed.* North-Holland.

Kuehn, C. (2019). *PDE dynamics. An Introduction*. SIAM, Philadelphia.

Øksendal, B. (2003). *Stochastic Differential Equations*. Springer, 6th edition.

Olver, P. J. (2014). *Introduction to Partial Differential Equations*. Undergraduate Texts in Mathematics. Springer International Publishing, 1 edition.

Pavliotis, G. A. (2014). *Stochastic Processes and Applications: Diffusion Processes, the Fokker-Planck and Langevin Equations*. Texts in Applied Mathematics 60. Springer-Verlag New York, 1 edition.

Perko, L. (1996). *Differential Equations and Dynamical Systems, 2nd Ed.* Springer-Verlag.

Pham, H. (2009). *Continuous-time Stochastic Control and Optimization with Financial Applications*. Stochastic Modelling and Applied Probability. Springer, 1 edition.

Salsa, S. (2016). *Partial Differential Equations in Action: From Modelling to Theory*. Number 99 in Unitext. Springer International Publishing, third edition.

Seierstad, A. (2009). *Stochastic control in discrete and continuous time*. Springer.

Stokey, N. L. (2009). *The Economics of Inaction*. Princeton.

Turnovsky, S. (1995). *Methods of Macroeconomic Dynamics*. MIT Press.

Weber, T. A. (2011). *Optimal Control Theory with Applications in Economics*. The MIT Press.

Other references will be given along the way and would be cited in the class-notes.

Software

Although this is not a course in numerical methods, the use of computers helps a lot in illustrating the solutions, solving, studying the dynamic properties, and estimating the models.

Useful software for solving differential equations:

- public license:
  - specialized for ODE's: auto (`http://indy.cs.concordia.ca/auto/`, `http://www.dam.brown.edu/people/sandsted/homcont.php`), and xpp (`http://www.math.pitt.edu/~bard/xpp/xpp.html`)
  - generic languages: python (`https://www.python.org/` and for ODE's `https://docs.scipy.org/doc/scipy/reference/integrate.html`), R (`https://www.r-project.org/` and an example for solving ODE's `https://cran.r-project.org/web/packages/sundialr/vignettes/my-vignette.html` or `https://cran.r-project.org/web/packages/deSolve/index.html`), sagemath (`http://www.sagemath.org/`)
- proprietary: Mathematica (`https://www.wolfram.com/mathematica/`), Maple (`https://maplesoft.com/`, Matlab (`https://www.mathworks.com/products/matlab.html`).